

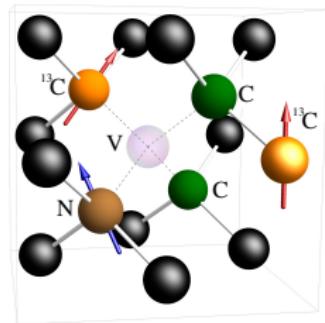
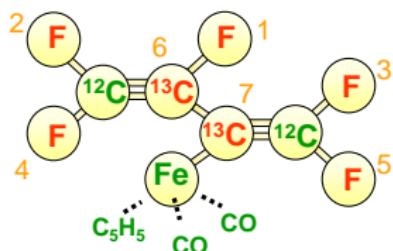
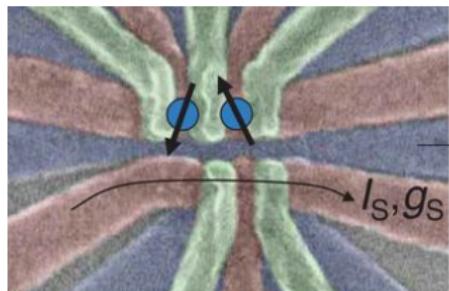
# Quantum computing

## Lecture 3

Hélène Perrin

February 7, 2021

# Lecture 3: More platforms for quantum computation



# Outline

Electron spins in quantum dots

- Coulomb blockade through quantum dots

- Double quantum dot

- Quantum dots in large  $B$  field: using the spin

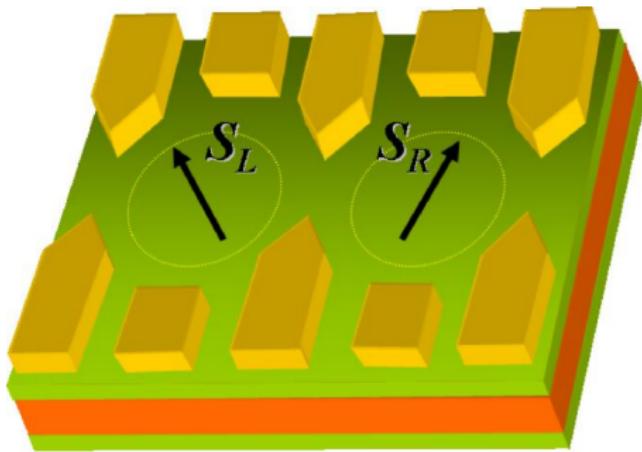
- Realizing a C-NOT gate

Other platforms

- NMR in molecules

- NV centers

# Electron spins in quantum dots



Loss & DiVincenzo proposal PRA **57**, 120 (1998)

# Electron spins in quantum dots for quantum computing

General idea

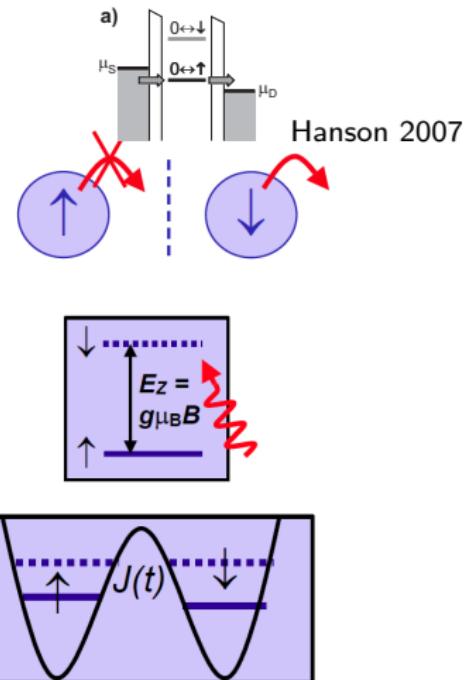
**Initialization** 1-electron, low  $T$ , high  $B_0$   
 $H_0 \sim \sum \omega_i \sigma_{zi}$

**Read-out** convert spin to charge  
 then measure charge

**ESR** pulsed microwave magnetic field  
 $H_{RF} \sim \sum A_i(t) \cos(\omega_i t) \sigma_{xi}$

**SWAP** exchange interaction  
 $H_J \sim \sum J_{ij}(t) \sigma_i \cdot \sigma_j$

**Coherence** long relaxation time  $T_1$   
 long coherence time  $T_2$

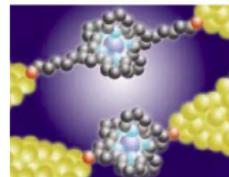


Slide: Lieven Vandersypen, TU Delft

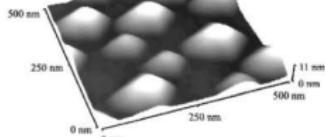
# Quantum dots gallery

From nano to micro devices

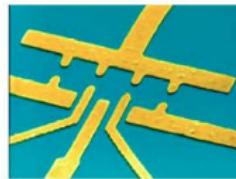
single molecule



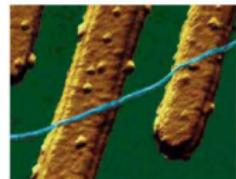
self-assembled QD



lateral QD



nanotube



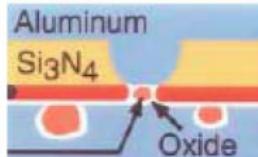
1 nm

10 nm

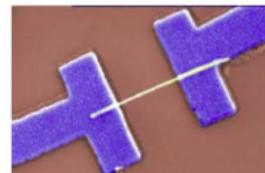
100 nm

1  $\mu$ m

metallic nanoparticle



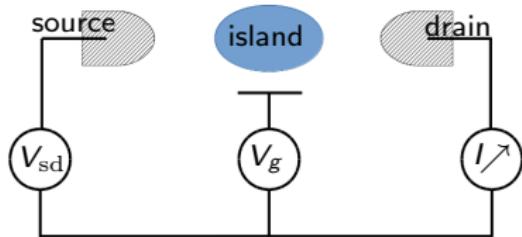
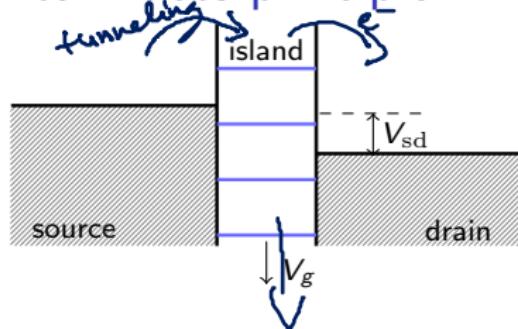
vertical QD



nanowire

Slide: Lieven Vandersypen, TU Delft

# Quantum dots principle



- tunneling through the thin barrier
- capacitive coupling

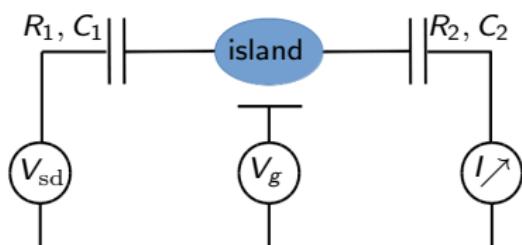
charging energy :

$$E_C = \frac{e^2}{2C} \quad C \approx 10^{-18} F$$

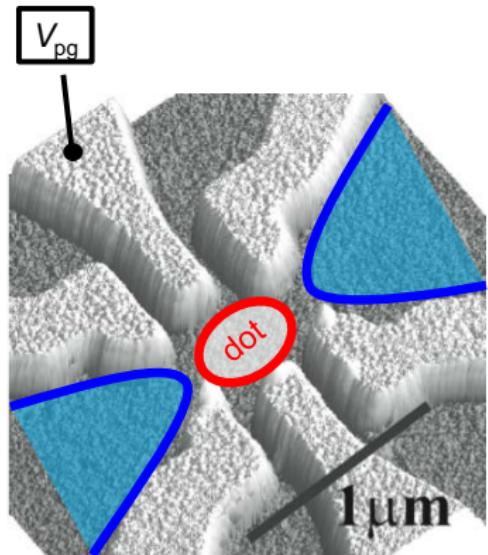
GaAs / AlGaAs initially

now : Si / SiO<sub>2</sub>  
or Si / Ge

to make it compatible with computer industry

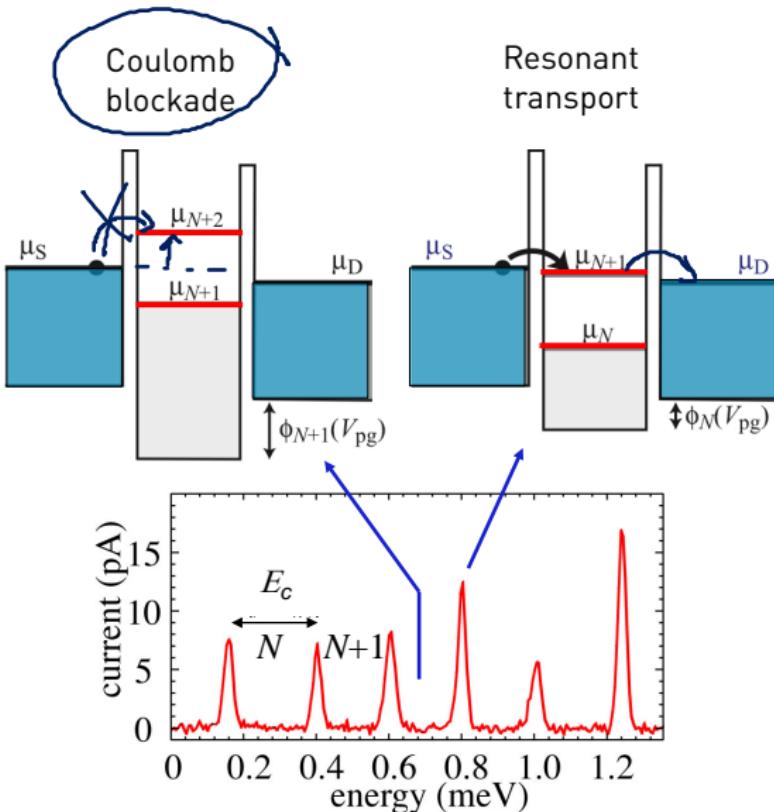


# Coulomb blockade

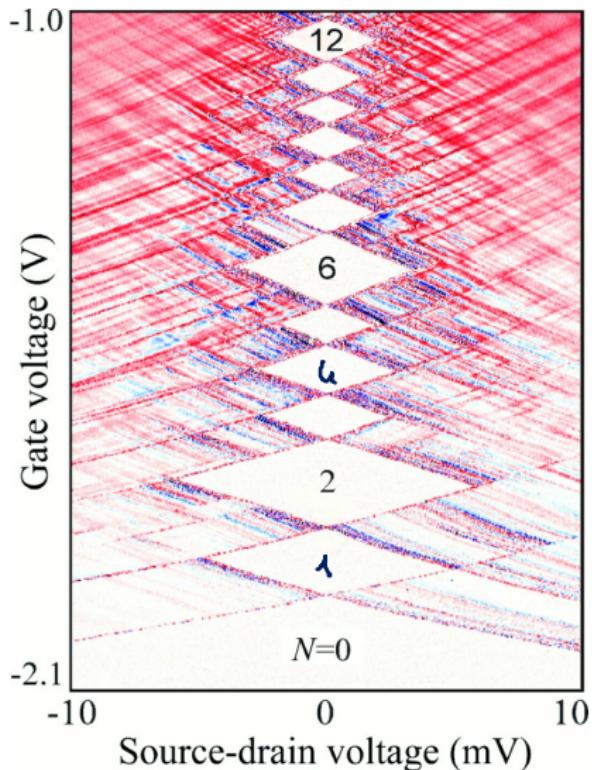


Quantum effects become visible at temperatures  $< 4.2\text{ K}$

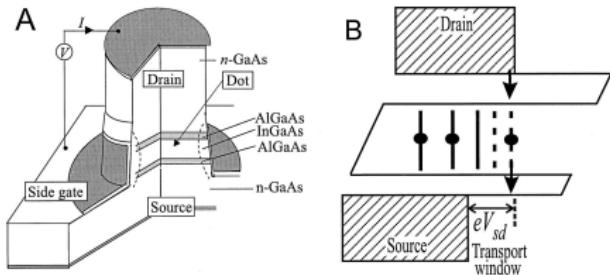
Slide: Thomas Ihn, ETH Zürich



# Current peak width: the Coulomb ‘diamonds’



Differential conductance through the dot



- ▶ Conductance window larger with a bias
- ▶ Very narrow at zero bias
- ▶ Larger diamonds when the electron orbitals fill a ‘shell’ in this 2D atom, i.e.  $N = 2, 6, 12 \dots$

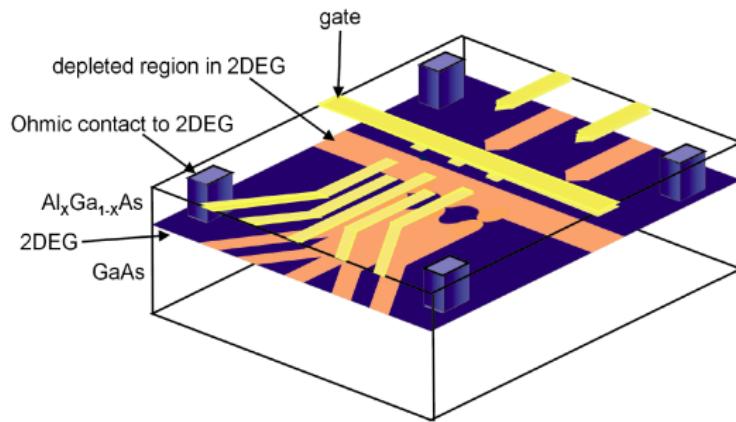


[Kouwenhoven et al. Science 278 1788 (1997)]

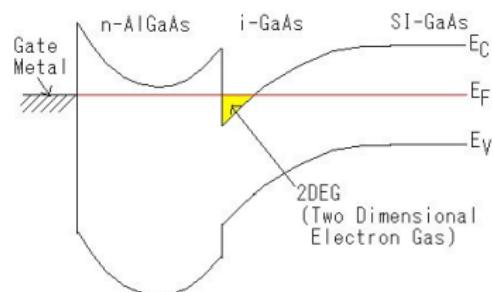
# Semi-conductor quantum dots

Electrically controlled parameters

Set of electrodes to shape / control the electronic density



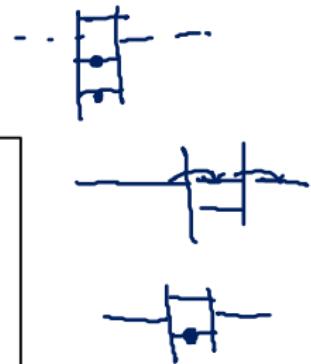
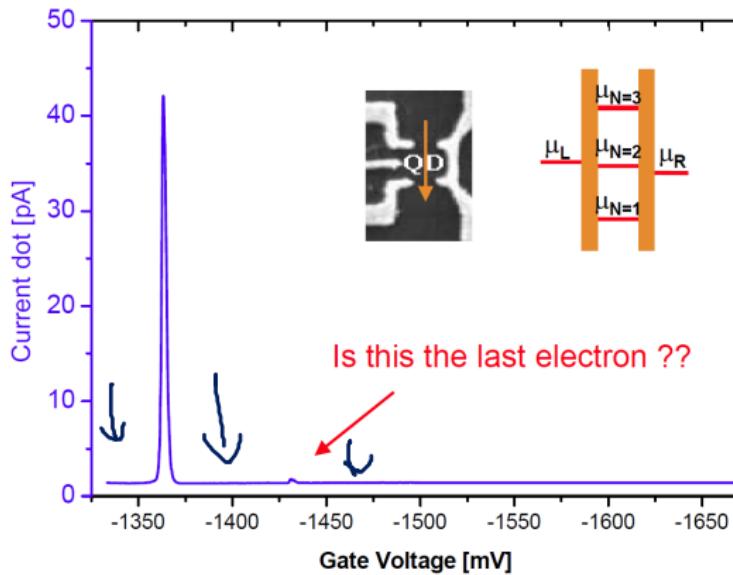
transverse confinement to a 2DEG



Slide: Lieven Vandersypen, TU Delft

# Transport through a quantum dot

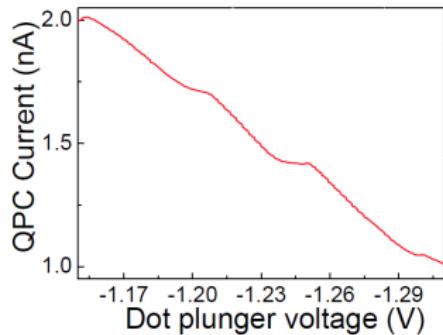
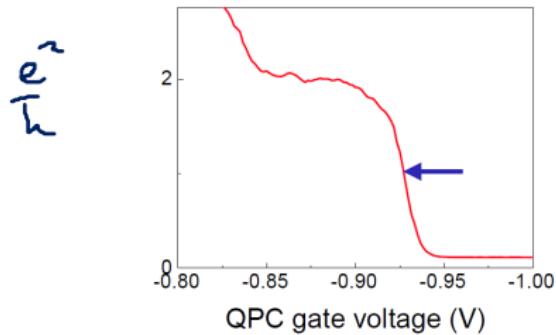
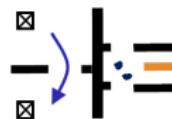
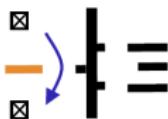
## Coulomb blockade



Slide: Lieven Vandersypen, TU Delft

# Add a quantum point contact as a charge detector

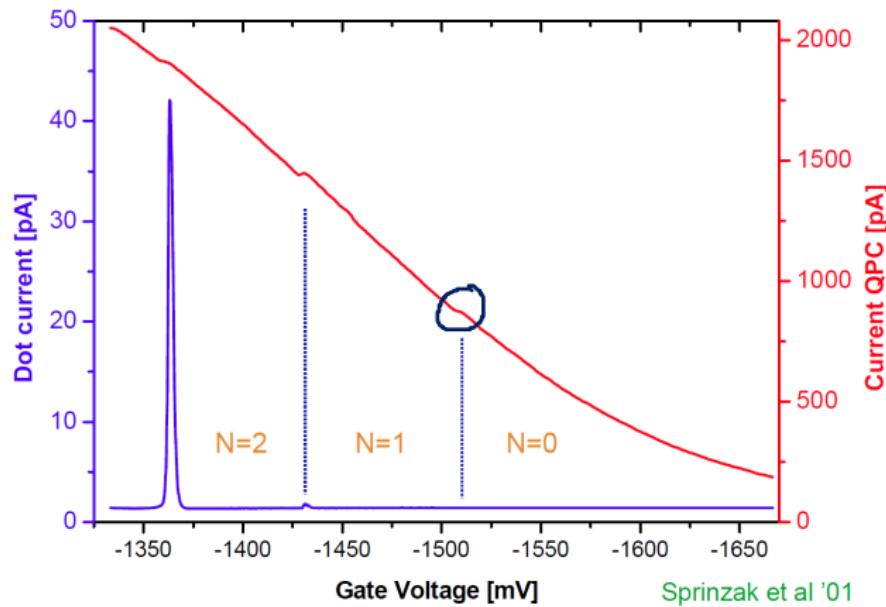
Conductance / current



Slide: Lieven Vandersypen, TU Delft

# Counting up to the last electron in the dot

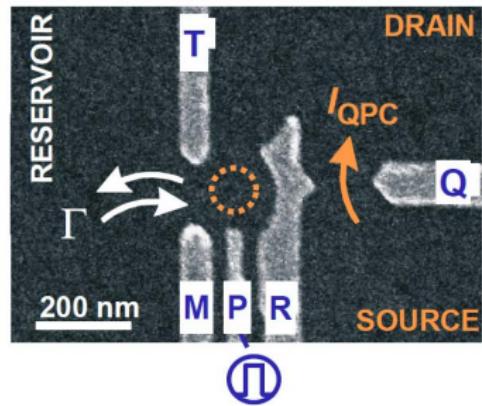
QPC as a charge detector



Slide: Lieven Vandersypen, TU Delft

# Observing electrons going through a dot in real time

Current change in the QPC

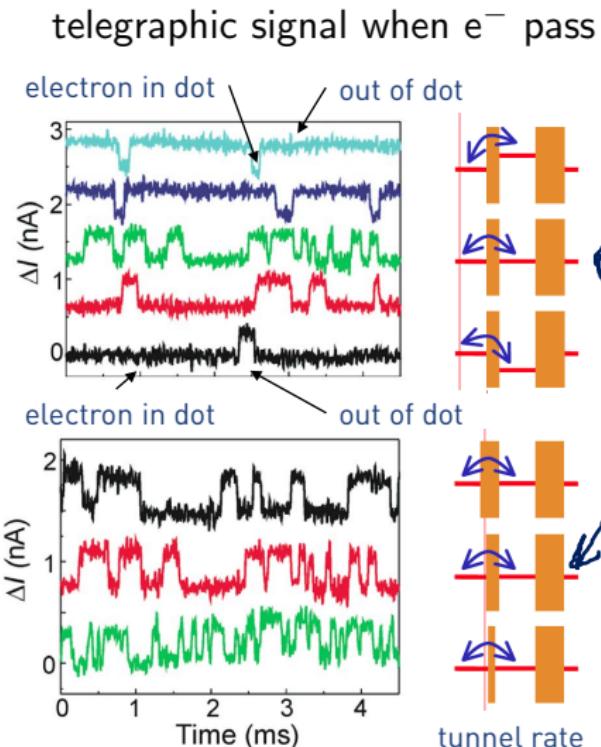


$$I_{QPC} \sim 30 \text{ nA}$$

$$\Delta I_{QPC} \sim 0.3 \text{ nA}$$

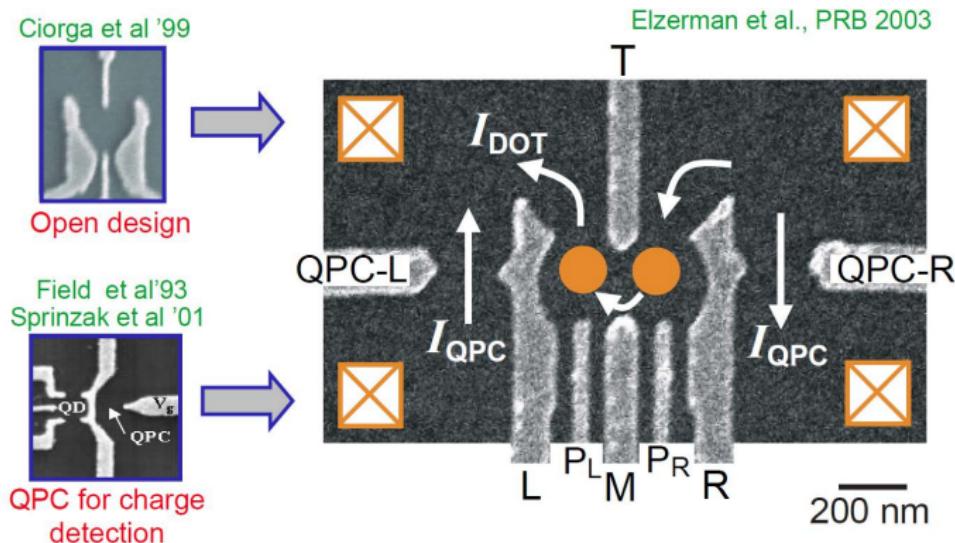
[Vandersypen et al., App. Phys. Lett.  
85, 4394 (2004)]

Slide: adapted from Lieven Vandersypen, TU Delft



# Double quantum dot

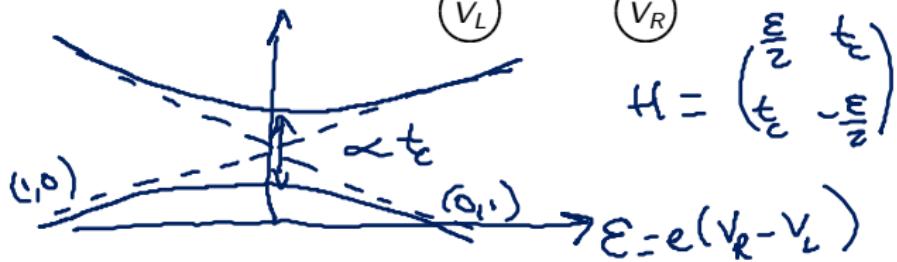
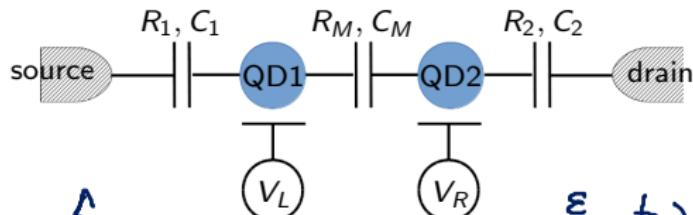
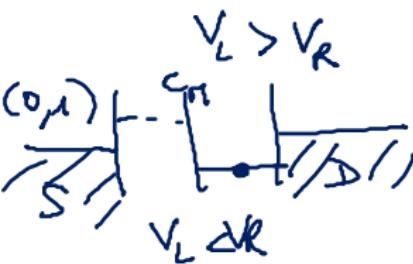
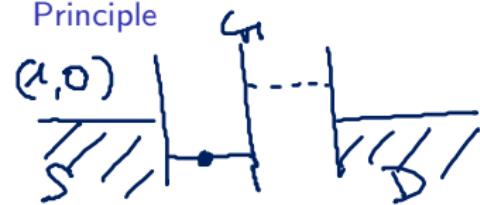
Coupling two qdots together



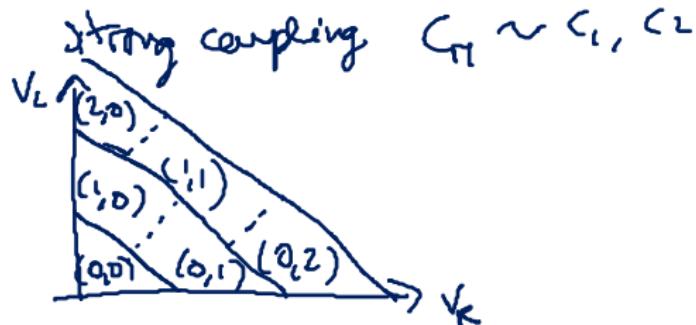
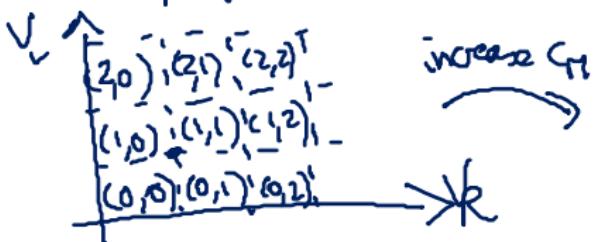
Slide: Lieven Vandersypen, TU Delft

# Double quantum dot

Principle

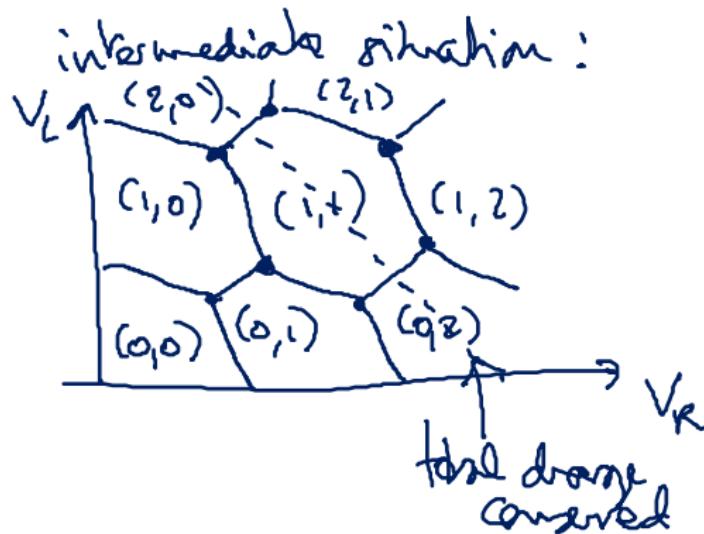


no coupling :  $C_{II} = 0$



# Double quantum dot

$V_L - V_R$  diagramme depending on the coupling  $C_M$



current non zero for  
 $V_{SD}=0$  only at the  
 triple points.

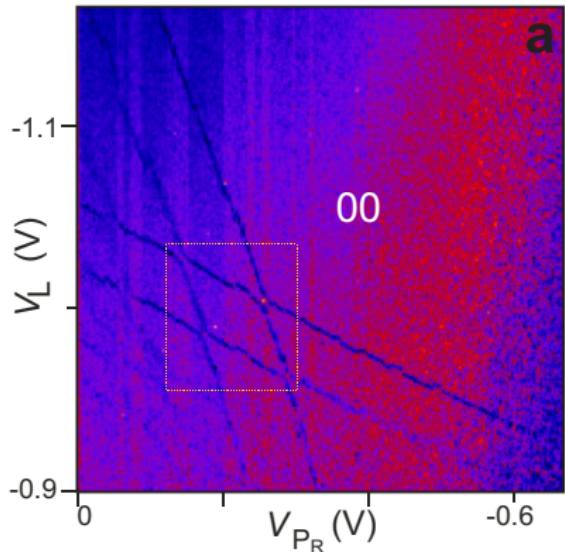
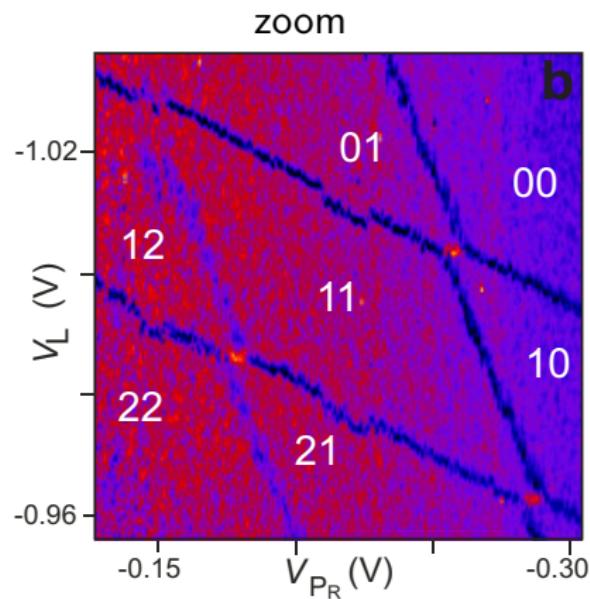
if  $V_{SD} \neq 0$ , the triple points open in triangle



# Double quantum dot

Differential conductance spectroscopy at  $B = 0$  [Elzerman et al., PRB **67**, 161308 (2003)]

$$dI_{QPC}/dV_L$$

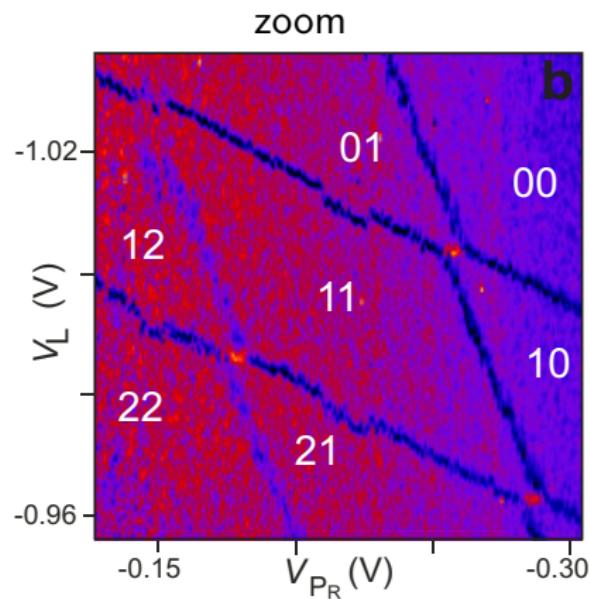
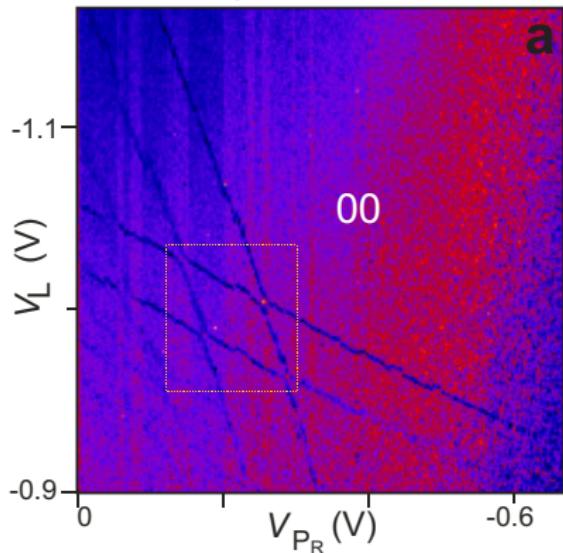
**a****zoom****b**

Use the  $|01\rangle$  and  $|10\rangle$  coupled states?

# Double quantum dot

Differential conductance spectroscopy at  $B = 0$  [Elzerman et al., PRB **67**, 161308 (2003)]

$$dI_{QPC}/dV_L$$

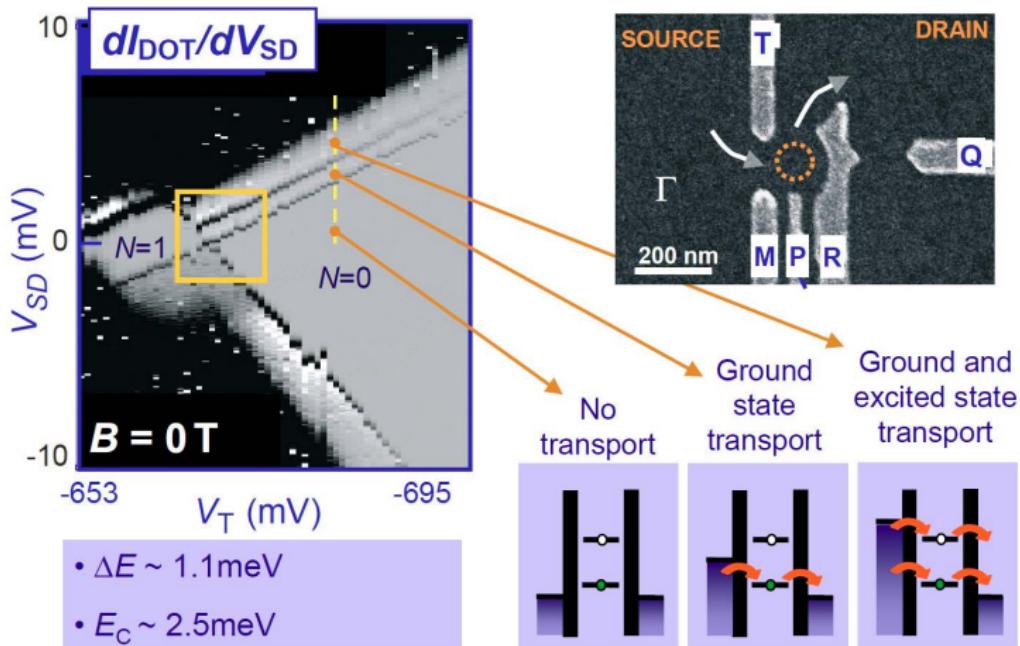


Use the  $|01\rangle$  and  $|10\rangle$  coupled states?

But charge relaxation is strong:  $T_1 \sim 16$  ns and  $T_2 < 1$  ns times **too short!** Use electronic **spin**.

# Energy levels at $B = 0$

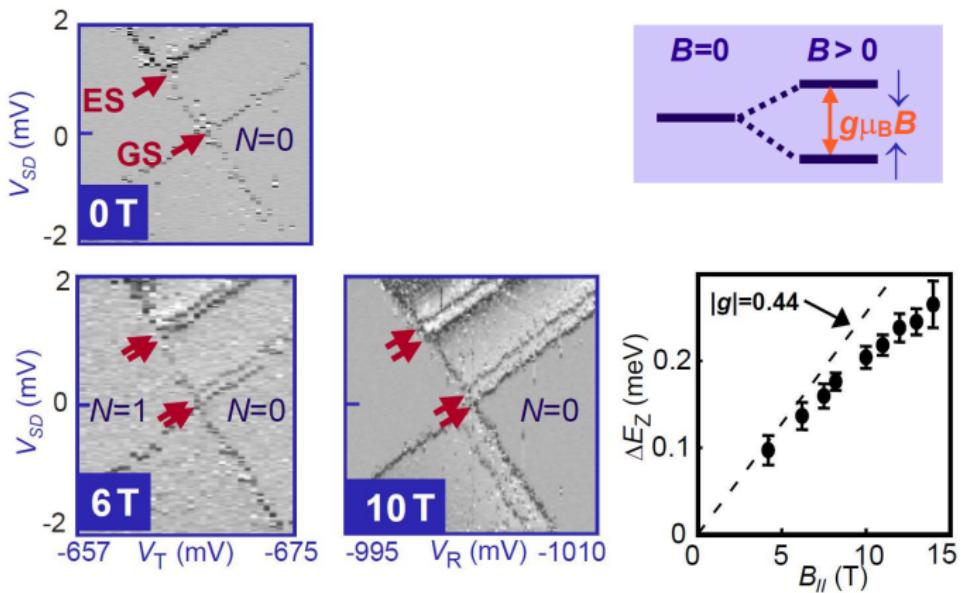
Single quantum dot



Slide: Lieven Vandersypen, TU Delft

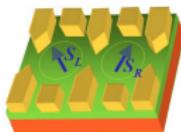
# Energy levels at $B \neq 0$

Single quantum dot



Slide: Lieven Vandersypen, TU Delft

# Quantum operations for spin qubits



Loss & DiVincenzo,  
PRA **57**, 120 [1998]  
Vandersypen *et al.*,  
Proc. MQC02(quant-ph/0207059)

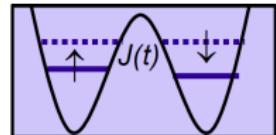
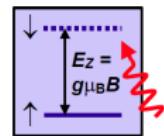
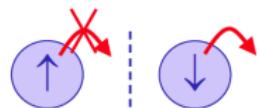
**Initialization** 1-electron, low  $T$ , high  $B_0$   
 $H_0 \sim \sum \omega_i \sigma_{zi}$

**Read-out** convert spin to charge  
then measure charge

**ESR** pulsed microwave magnetic field  
 $H_{RF} \sim \sum A_i(t) \cos(\omega_i t) \sigma_{xi}$

**SWAP** exchange interaction  
 $H_J \sim \sum J_{ij}(t) \sigma_i \cdot \sigma_j$

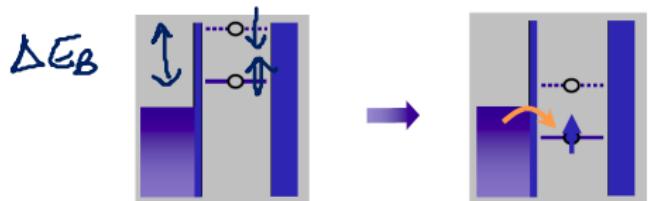
**Coherence** long relaxation time  $T_1$   
long coherence time  $T_2$



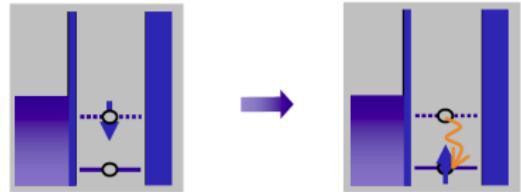
Slide: Lieven Vandersypen, TU Delft

# Initialization

Method 1:  
spin-selective tunneling

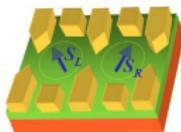


Method 2:  
relaxation to ground state



Slide: Lieven Vandersypen, TU Delft

# Readout



Loss & DiVincenzo, *PRA* 1998  
 Vandersypen et al., *Proc. MQC02* (quant-ph/0207059)

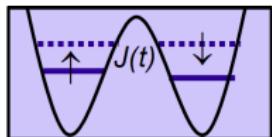
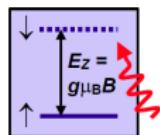
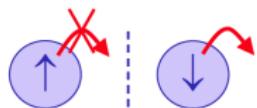
**Initialization** 1-electron, low  $T$ , high  $B_0$   
 $H_0 \sim \sum \omega_i \sigma_{zi}$

**Read-out** convert spin to charge  
 then measure charge

**ESR** pulsed microwave magnetic field  
 $H_{RF} \sim \sum A_i(t) \cos(\omega_i t) \sigma_{xi}$

**SWAP** exchange interaction  
 $H_J \sim \sum J_{ij}(t) \sigma_i \cdot \sigma_j$

**Coherence** long relaxation time  $T_1$   
 long coherence time  $T_2$

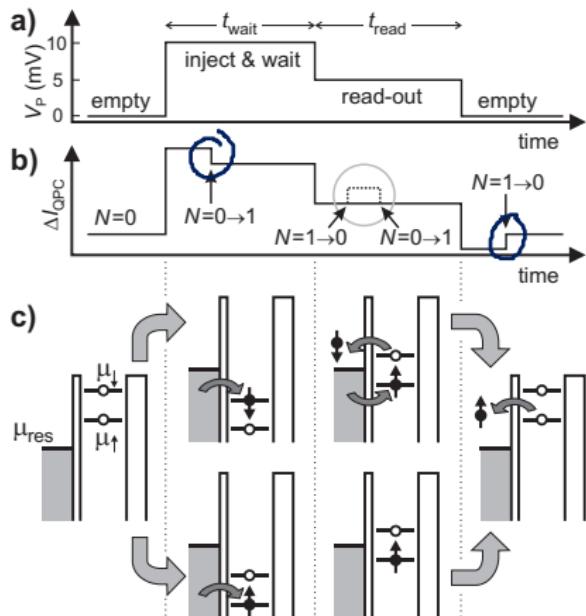


Slide: Lieven Vandersypen, TU Delft

# Spin state detection

Look for electron pulses through a QPC

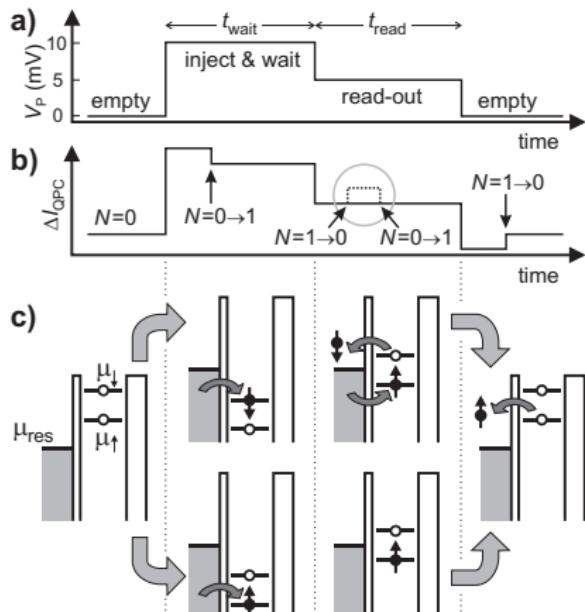
## Principle of measurement



# Spin state detection

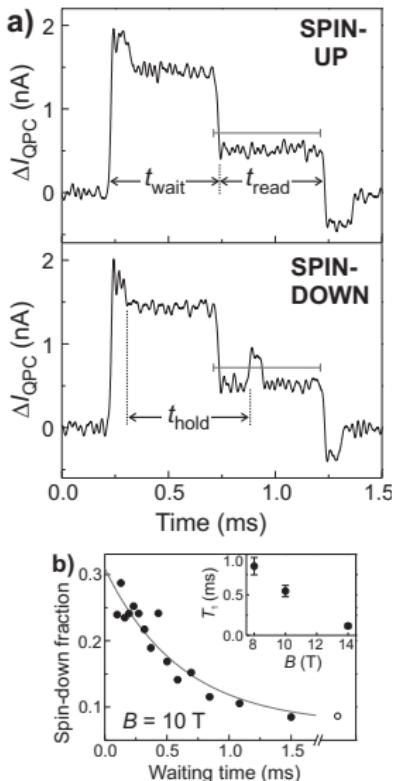
Look for electron pulses through a QPC

## Principle of measurement

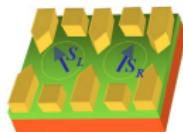


[Fig. from Hanson 2007]

## Results



# Single qubit gates with microwave driving



Loss & DiVincenzo, *PRA* 1998  
 Vandersypen et al., *Proc. MQC02* (quant-ph/0207059)

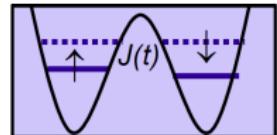
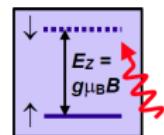
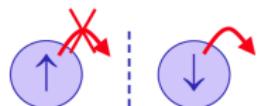
**Initialization** 1-electron, low  $T$ , high  $B_0$   
 $H_0 \sim \sum \omega_i \sigma_{zi}$

**Read-out** convert spin to charge  
 then measure charge

**ESR** pulsed microwave magnetic field  
 $H_{RF} \sim \sum A_i(t) \cos(\omega_i t) \sigma_{xi}$

**SWAP** exchange interaction  
 $H_J \sim \sum J_{ij}(t) \sigma_i \cdot \sigma_j$

**Coherence** long relaxation time  $T_1$   
 long coherence time  $T_2$



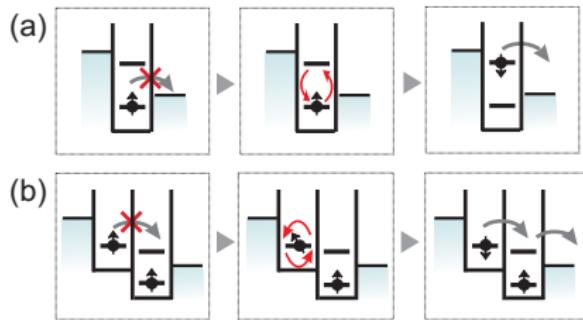
Slide: Lieven Vandersypen, TU Delft

# Electron spin resonance (ESR)

Observation of Zeeman splitting

Coupling term: **spin-orbit coupling**

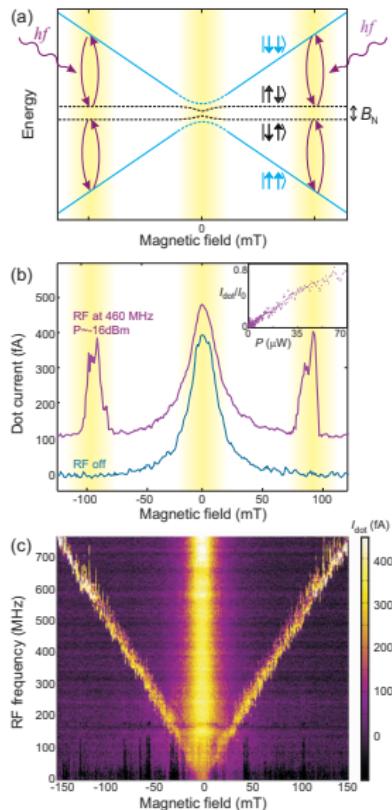
## Principle of ESR



Use Coulomb or spin blockade to isolate the two-level system

[Fig. from Hanson 2007]

## Results

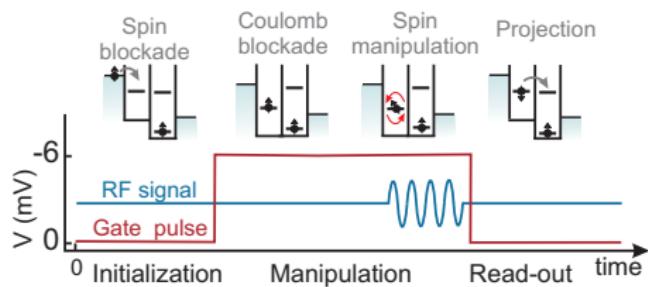


# Electron spin resonance (ESR)

Observation of Rabi oscillations

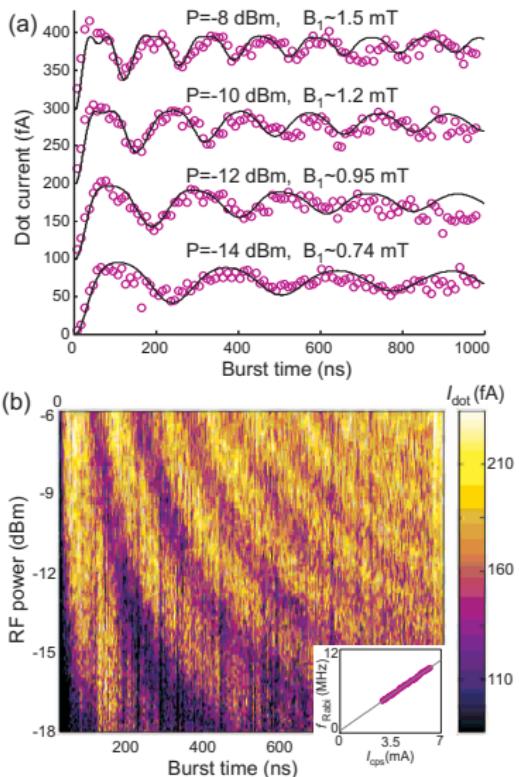
Coupling term: **spin-orbit coupling**

## Experimental sequence

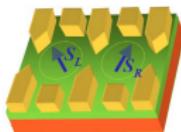


[Fig. from Hanson 2007]

## Results



# Two-qubit gates through exchange interaction



Loss & DiVincenzo,  
PRA **57**, 120 [1998]  
Vandersypen *et al.*,  
Proc. MQC02(quant-ph/0207059)

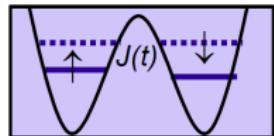
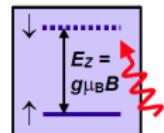
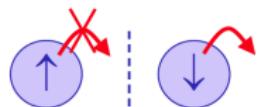
**Initialization** 1-electron, low  $T$ , high  $B_0$   
 $H_0 \sim \sum \omega_i \sigma_{zi}$

**Read-out** convert spin to charge  
then measure charge

**ESR** pulsed microwave magnetic field  
 $H_{RF} \sim \sum A_i(t) \cos(\omega_i t) \sigma_{xi}$

**SWAP** exchange interaction  
 $H_J \sim \sum J_{ij}(t) \sigma_i \cdot \sigma_j$

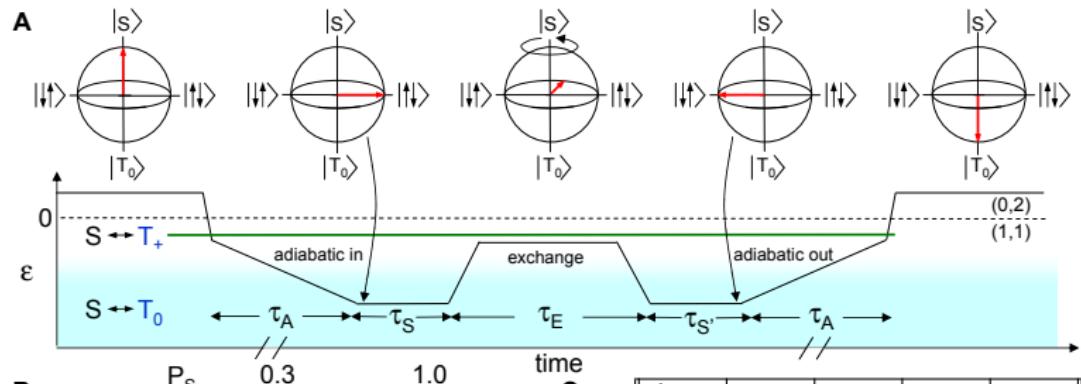
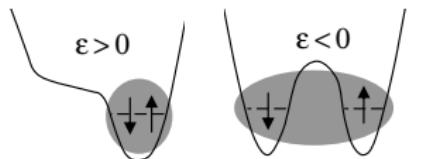
**Coherence** long relaxation time  $T_1$   
long coherence time  $T_2$



Slide: Lieven Vandersypen, TU Delft

# Two qubit gate: exchange interaction

Coupling term: **spin exchange**  $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$   
 Couples  $|S\rangle \propto |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$  and  $|T_0\rangle \propto |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

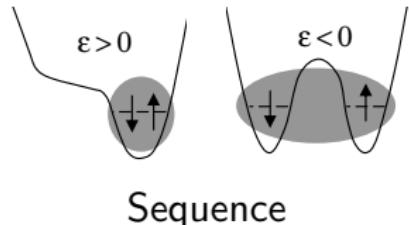
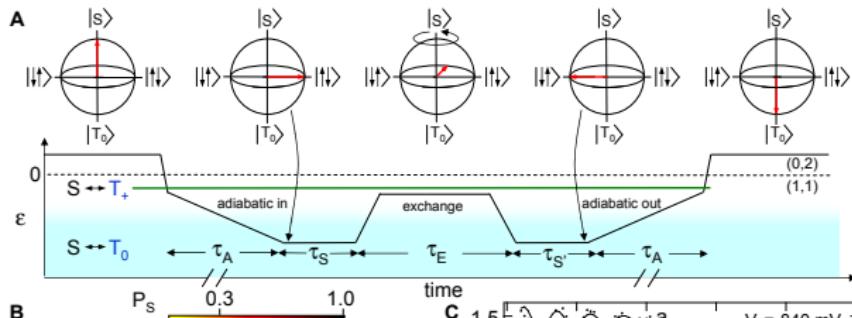


$$\begin{aligned} T_+ &= |\uparrow\downarrow\rangle \\ T_0 &= |\downarrow\uparrow\rangle \end{aligned}$$

# Two qubit gate: exchange interaction

Coupling term: **spin exchange**  $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$

Couples  $|S\rangle \propto |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$  and  $|T_0\rangle \propto |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

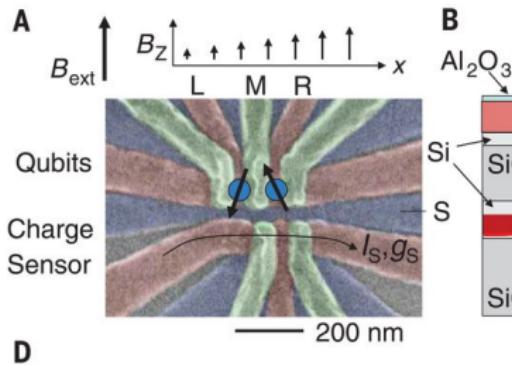


- ▶ bring singlet  $S$  and triplet  $T_0$  state in resonance
- ▶ adjust coupling with imbalance  $\varepsilon$
- ▶ free evolution under exchange Hamiltonian
- ▶  $\sqrt{\text{SWAP}}$  in 180 ps

[Fig. from Hanson 2007]

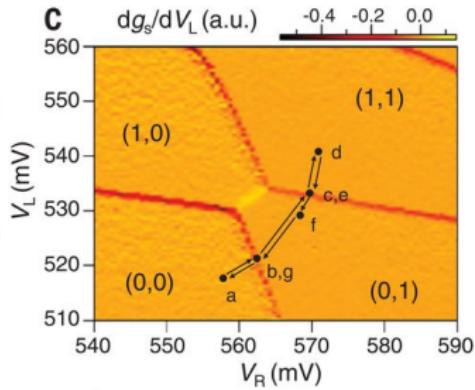
# Realizing a C-NOT gate

## Experimental device and sequence



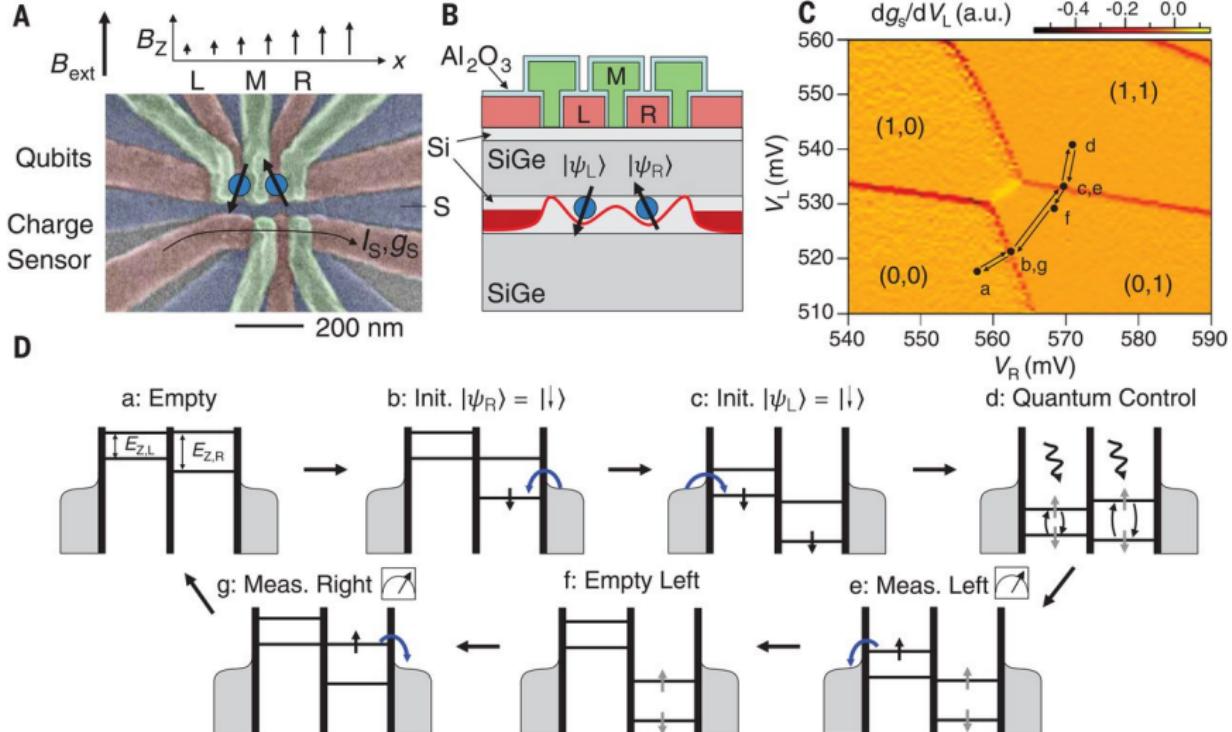
- ▶ magnetic gradient  $\Rightarrow$  different/addressable frequency for L/R spins
- ▶ adjust  $N_L$  and  $N_R$  with  $V_L$  and  $V_R$  gates
- ▶ adjust tunnel coupling between wells with  $V_M$  gate
- ▶ perform spin exchange gates in (1,1) domain

[Fig. from Zajac et al. Science 359, 439 (2018)]



# Realizing a C-NOT gate

## Experimental device and sequence

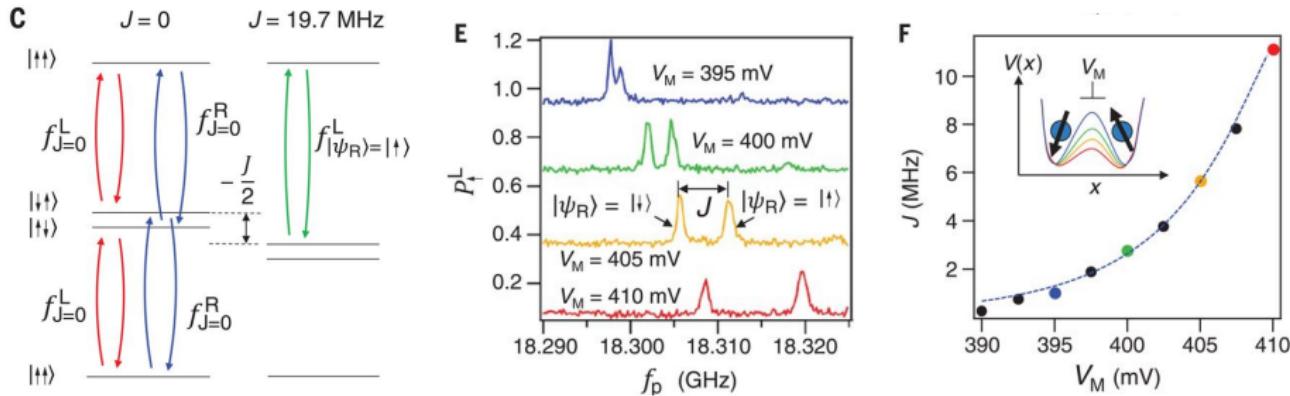


[Fig. from Zajac et al. Science 359, 439 (2018)]

# Realizing a C-NOT gate

## Tuning the coupling

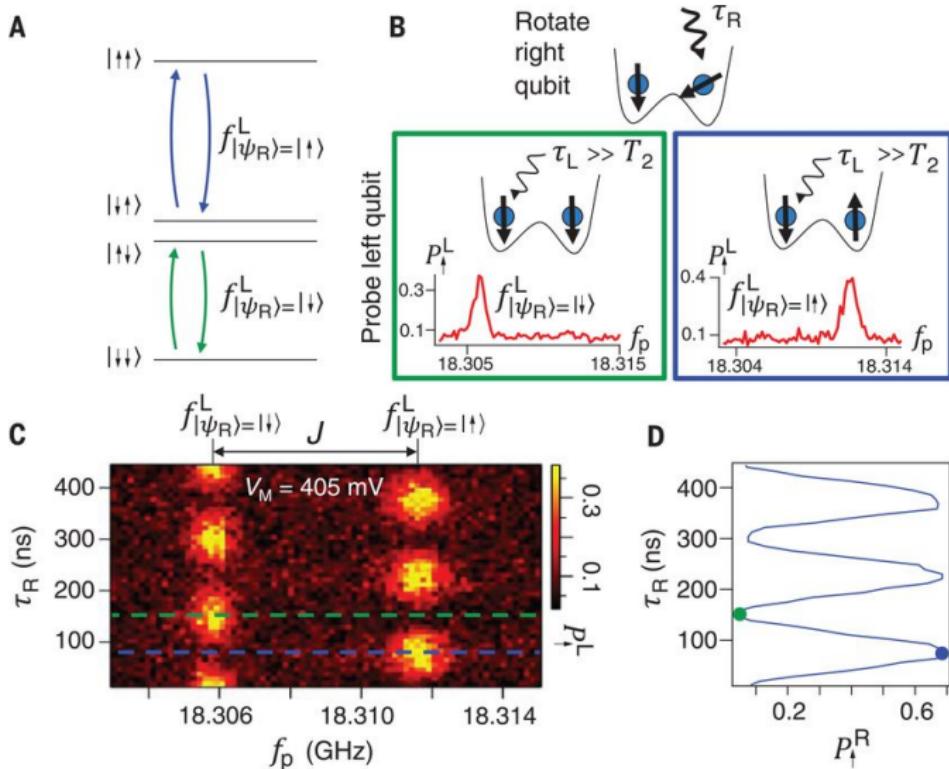
The states  $|\uparrow\downarrow\rangle$  and  $|\downarrow\uparrow\rangle$  are shifted due to exchange coupling  $J$ . This makes the  $|\uparrow X\rangle \rightarrow |\downarrow X\rangle$  transition depend on  $X = \uparrow, \downarrow$ .  $J$  is tuned through the  $V_M$  middle gate.



[Fig. from Zajac et al. Science 359, 439 (2018)]

# Realizing a C-NOT gate

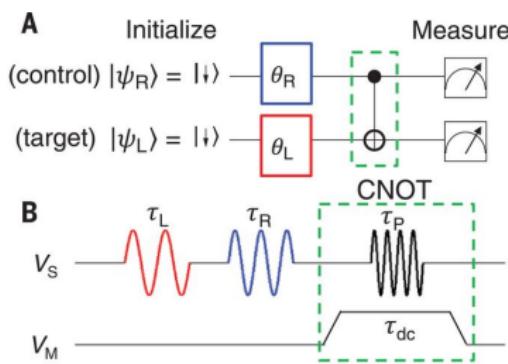
Resonant frequency of L depends on spin state of R [Zajac et al. Science 359, 439 (2018)]



# Realizing a C-NOT gate

## Results

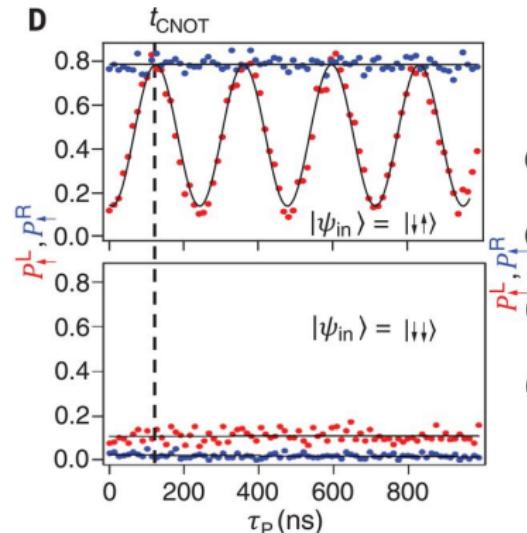
### Experimental sequence



CNOT gate obtained for  $\tau_P = t_{\text{CNOT}}$

[Fig. from Zajac et al. Science 359, 439 (2018)]

### Observation of conditional oscillations

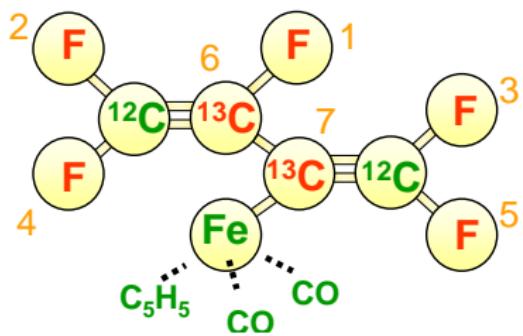


# Summary and recent trends on spin quantum dots

- ▶ A younger system, rapidly improving
- ▶ Fast gates (CNOT in 130 ns, single-qubit gates in about 100 ns)
- ▶ Best single-qubit gate fidelity > 99.9%; Bell-state with > 80% fidelity; two-qubit gate ~ 95%
- ▶ Limitations: readout fidelity/decoherence
- ▶ Cavity mediated coupling also demonstrated [Burkard et al. (2019)]
- ▶ Current trends: all in (purified) Si

# NMR in molecules

Ex: molecule with 7 qubits = nuclear spins



Vandersypen et al., Nature 414, 883  
(2001)

- ▶ Qubits = nuclear spins
- ▶ Initialization: thermal distribution
- ▶ Single qubit gates: NMR spin rotation
- ▶ Very long coherence time  $\sim 1$  s
- ▶ Coupling between qubits: direct spin-spin interaction. Rotation frequency depends on orientation of neighbouring qubit
- ▶ Shor algorithm  $15 = 3 \times 5$  implemented in 2001
- ▶ But not scalable...

# NV centers in diamond

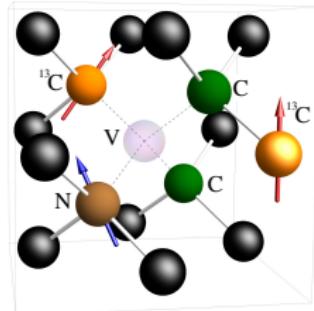
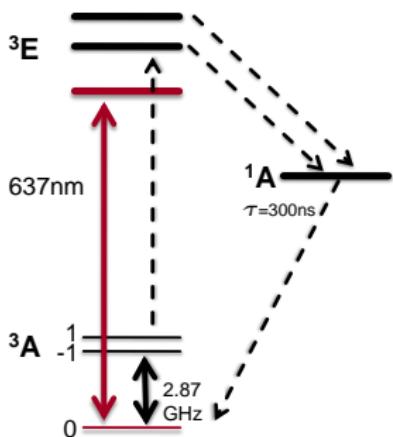


Fig: Doherty et al.



- ▶ Defect in diamond: vacancy + nitrogen substitution
- ▶ Optical frequency (637 nm) for state preparation (pumping)
- ▶ Readout with ion-like technique (dark / bright state)
- ▶ Hyperfine structure 2.87 GHz, magnetic dependent, for single-qubit rotation,  $T_2 \sim 1 \text{ ms}$
- ▶ 4 possible orientations of the NV center
- ▶ Two-qubit gates using surrounding nuclear spins

Level scheme: Jörg Wrachtrup, Stuttgart

# Bibliography for Lecture 3

- ▶ G. Benenti, G. Casati and G. Strini, *Principles of Quantum Computation and Information* vol. 2, World Scientific 2007
- ▶ Lecture notes and teaching material by Andreas Walraff  
<https://qudev.phys.ethz.ch/node/2070>
- ▶ R. Hanson et al., *Spins in few-electron quantum dots*, Rev. Mod. Phys. **79**, 1217 (2007)
- ▶ G. Burkard et al., *Superconductor-semiconductor hybrid cavity quantum electrodynamics* arXiv <https://arxiv.org/abs/1905.01155>
- ▶ M. W. Doherty et al., *The nitrogen-vacancy colour centre in diamond*, Physics Reports **528**, 1 (2013), <https://arxiv.org/abs/1302.3288>