

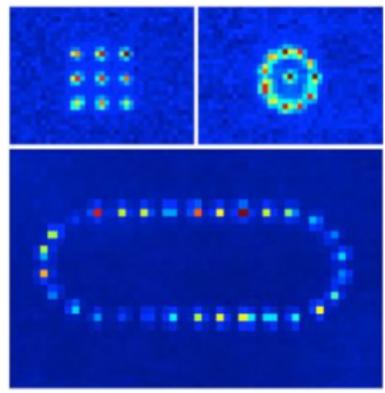
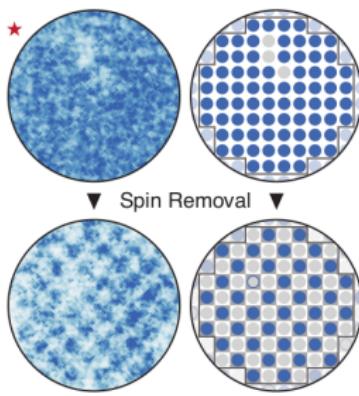
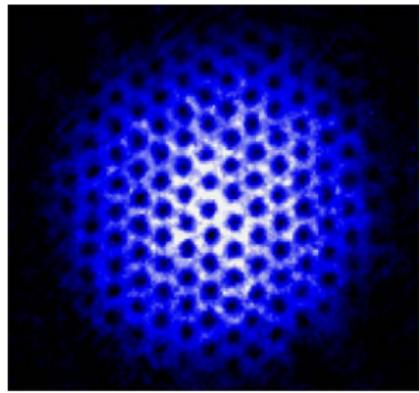
Quantum computing

Lecture 4

Hélène Perrin

February 11, 2021

Lecture 4: Atomic platforms for quantum simulation



Outline

Introduction

Quantum gases as quantum simulators

Basics of quantum gases

Experimental methods

A widely tunable system

First demonstrations of quantum simulation with ultracold atoms

Rydberg atoms in optical tweezers

Properties of Rydberg atoms

Dipole-dipole interactions between Rydberg atoms

Many-body physics with Rydberg atoms

What is quantum simulation?

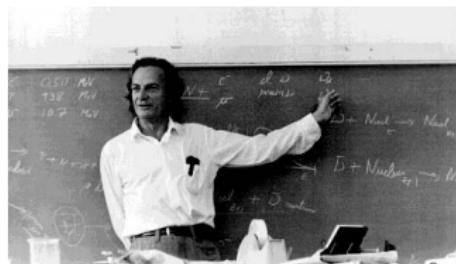
Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

International Journal of Theoretical Physics



Key idea: classical computers are **inefficient** to describe N interacting particles ($N \gg 1$)

Hilbert space grows **exponentially** with N

Proposed solution:

Build a well-controlled system which would realize a **given Hamiltonian**

Directly measure its properties: ground state, excitation spectrum...

⇒ **non universal** simulator, but **more accessible** than a universal quantum computer quantique universel.

Relevant topics to be simulated

- ▶ **Equilibrium quantum systems, bulk or lattice**

Phase diagrams, equation of state, superconductivity and spin imbalance, superfluidity

- ▶ **Out-of-equilibrium systems and quantum quenches**

Transport and dissipation, Kibble-Zurek scenario, many-body localization

- ▶ **Quantum magnetism**

Individual particle detection, lattices systems, frustration, impurity problem

- ▶ **Topological systems**

Quantum Hall effect, spin-orbit coupling, gauge fields, Majorana fermions, link with quantum computation

- ▶ **Simulation of lattice gauge theories**

Abelian or non-Abelian Higgs mechanism

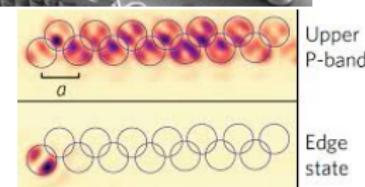
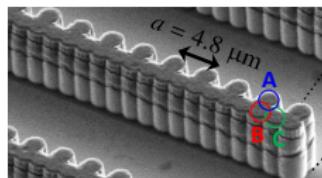
- ▶ **Relevant theory**

Preparation, measurements, dissipation and entanglement

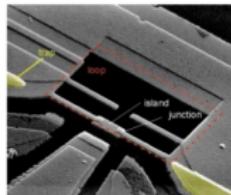
Thermalization of isolated Q-systems, quenches, entanglement growth

Platforms for quantum simulation

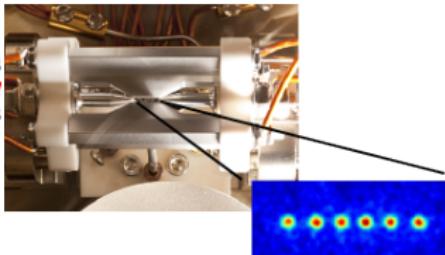
Examples of possible experimental platforms:



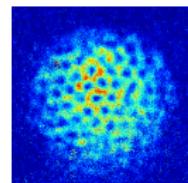
polaritons



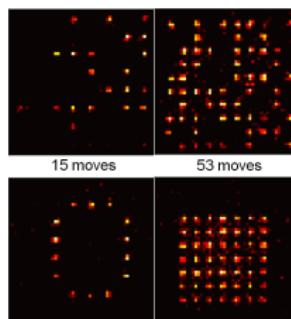
superconductors



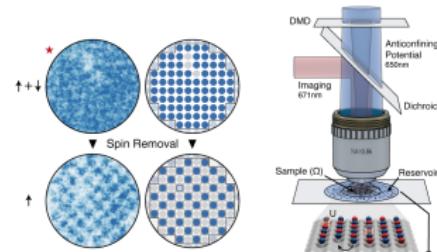
trapped ions



quantum gases (bulk)

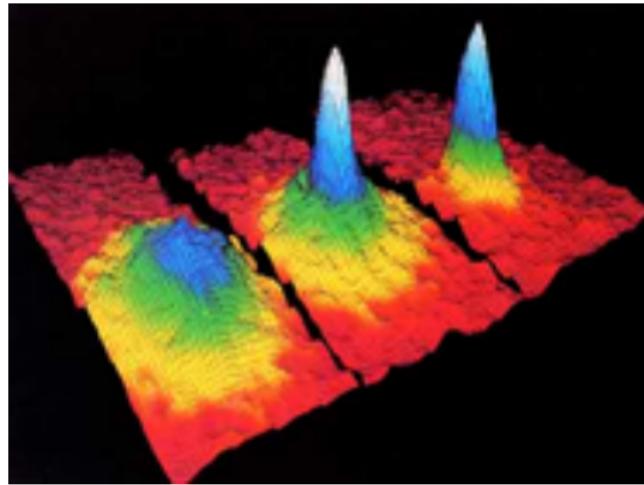


Rydberg atoms



quantum gases in lattices

Basics of quantum gases



Cornell & Wieman (1995)

1995: first Bose-Einstein condensates of dilute gases



Rb

E. Cornell



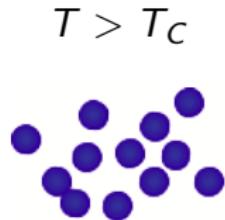
C. Wieman



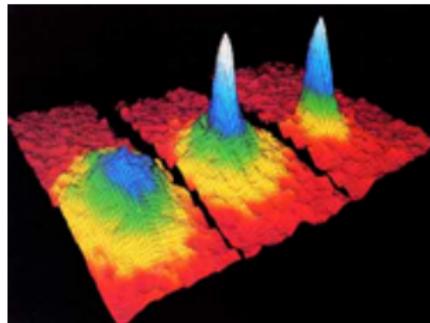
Na

W. Ketterle

Nobel prize 2001



$$T > T_c$$



$$T < T_c$$

Bose-Einstein condensates and degenerate Fermi gases

Gallery of quantum gases

Periodic table of quantum gases																	
alkali			transition metals												rare gases		
1998	H																
1999	Li	Be															
1995	Na	Mg															
2001	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br
1996	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Se	Te	I
2002	Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At
	Fr	Ra	Ac														
2001																	
2011																	
2012																	
2003																	
lanthanides																	
Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb Lu																	
Th Pa U Np Pu Am Cm Bk Cf Es Em Md No Lr																	

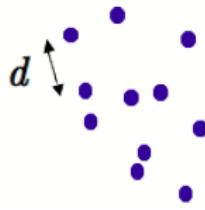
- + molecular BECs with: two fermions (K_2 , Li_2 , Er_2 , $LiK\dots$), two bosons (Cs_2 , $RbCs\dots$)
- + or molecular Fermi gases with fermion+boson (KRb , $RbCs$, $LiK\dots$)

Critical temperature



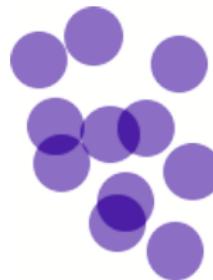
Criterion for degeneracy: more than one atom per state.

Size of a state in phase space: $\lambda_T = \frac{h}{\sqrt{2\pi m k_B T}}$ de Broglie wavelength



$$T > T_c$$

$$\lambda_T \ll d$$



$$T \sim T_c$$

$$\lambda_T \sim d$$



$$T < T_c$$

$$\lambda_T > d$$

all particles in the same
wavefunction?

Critical temperature

Saturation of the excited states

With a density of state (DOS) of the form

$$3D \text{ free-trap } D=3 \quad \delta = \frac{3}{2}$$

$$\rightarrow k = 2$$

$$3D \text{ in a box : } \delta = 0$$

$$\rightarrow k = \frac{1}{2}$$

where ε_0 is the energy of the nondegenerate ground state.

Ex: power law trap $U(r) = Cr^{D/\delta}$ in dimension D : $k = \delta + \frac{1}{2}$.

$$\boxed{\begin{aligned} f(\varepsilon) &= \frac{1}{e^{\beta(\varepsilon-\mu)} - 1} = \frac{1}{z^{-1} e^{\beta(\varepsilon-\varepsilon_0)} - 1} & z &= e^{\beta(\mu-\varepsilon_0)} < 1. \\ && \mu < \varepsilon & \mu \rightarrow \varepsilon \\ && z \rightarrow 1 & \end{aligned}}$$

Atom number in the excited states:

$$N' = \int_{\varepsilon_0}^{+\infty} f(\varepsilon) \rho(\varepsilon) d\varepsilon = \mathcal{A} \Gamma(k+1) (k_B T)^{k+1} g_{k+1}(z)$$

$g_{k+1}(z) < g_{k+1}(1)$ where

$$g_{k+1}(z) = \sum_{n=1}^{+\infty} \frac{z^n}{n^{k+1}}.$$

Polylog function

Critical temperature

Saturation of the excited states

$$g_{k+1}(1) = \sum_{n=1}^{\infty} \frac{1}{n^{k+1}}$$

The atom number in the excited states is bounded from above: $\underline{z} < 1$

$$N' < N'_{\max}(T) = A\Gamma(k+1)(k_B T)^{k+1} g_{k+1}(1)$$

$g_{k+1}(1)$ is finite if $k > 0$. Then $N' < N'_{\max}(T)$ is bounded from above. If we define T_c such that $N'_{\max}(T_c) = N$, we get

$$\frac{N_0}{N} > 1 - \left(\frac{T}{T_c} \right)^{k+1}$$

$$T_c \propto N^{\frac{1}{k+1}}.$$

$$N'_{\max}(T) < N'_{\max}(T_c) = N$$

gas in a 3D box: $k = 3/2$, $n' \lambda_T^3 < g_{3/2}(1) = 2.612$

1D, 2D: no BEC

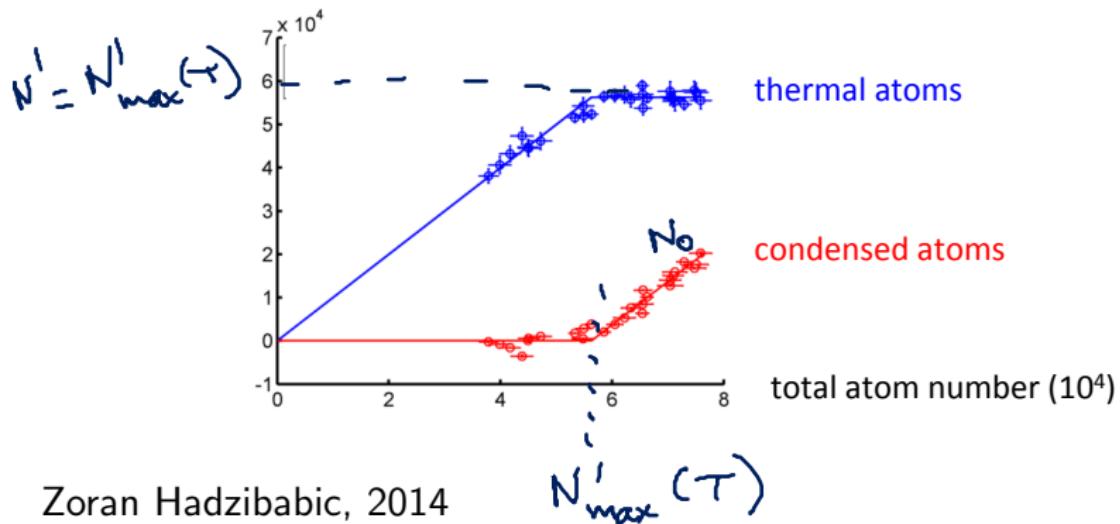
gas in a trap $U(\mathbf{r})$: $n'(\mathbf{r}) \lambda_T^D = g_{D/2} (e^{-\beta[U(\mathbf{r}) - \mu]})$

\Rightarrow upper bound $n'(0) \lambda_T^D < g_{D/2}(1)$

Critical temperature

Saturation of the excited states

Observation of the saturation of the excited states in a uniform gas (3D box trap).



Zoran Hadzibabic, 2014

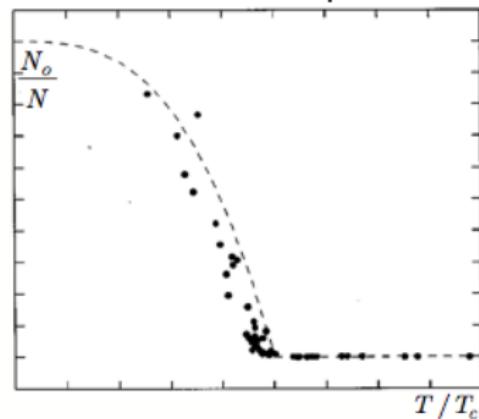
Critical temperature

Measurement of the condensate fraction in an harmonic trap

$$\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$$

$$k_B T_C = \hbar \bar{\omega} N^{1/3} \gg \hbar \bar{\omega}$$

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_C} \right)^3$$



Ensher et al., PRL 77, 4984 (1996)

Orders of magnitude: $N = 10^6$, $\omega_0/2\pi = 50 \text{ Hz}$
 $\implies T_C = 250 \text{ nK}$ **ultracold!**

Phase coherence



- ▶ weak interactions (dilute gas)
- ▶ the same wavefunction for all atoms

interferences between condensates

coherence length = cloud size

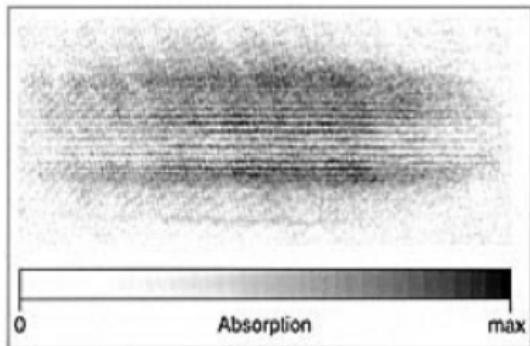
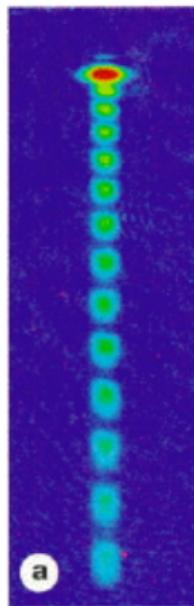


photo MIT 1996

photo
Munich
2000 →



Role of interactions

$$\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N) = \bigotimes_{i=1}^N \psi(r_i)$$


- ▶ weak interactions (dilute gas)
- ▶ the same wavefunction for all atoms
- ▶ effect of interactions: mean field
- ▶ Gross-Pitaevskii equation

$$\left(\underbrace{-\frac{\hbar^2 \nabla^2}{2m}}_{E_{\text{cin}}} + \underbrace{V_{\text{ext}}(\mathbf{r})}_{E_{\text{pot}}} + \underbrace{g |\psi|^2}_{E_{\text{int}}} \right) \psi = \mu \psi$$

$g = \frac{4\pi\hbar^2 a}{m}$ coupling constant for the interactions
 a scattering length
 μ chemical potential



Hydrodynamics

$$g > 0 \quad n(\vec{r}, t) = |\Psi(\vec{r}, t)|^2$$

Equivalent formulation of GPE using $\psi(\mathbf{r}, t) = \sqrt{n(\mathbf{r}, t)} e^{i\theta(\mathbf{r}, t)}$

$$(1) \quad \left. \begin{array}{l} \partial_t n + \nabla \cdot (n \mathbf{v}) = 0 \end{array} \right\} \text{continuity equation}$$

$$(2) \quad m \partial_t \mathbf{v} = -\nabla \left(-\frac{\hbar^2}{2m} \underbrace{\frac{\Delta(\sqrt{n})}{\sqrt{n}}}_{\text{quantum pressure}} + \frac{1}{2} m \mathbf{v}^2 + V_{\text{ext}} + g n \right)$$

(2) Euler equation

quantum pressure

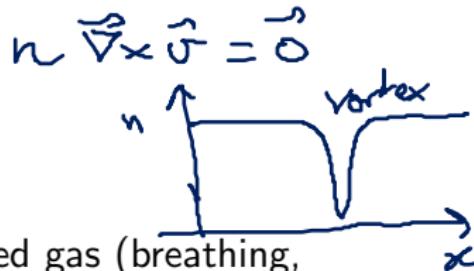
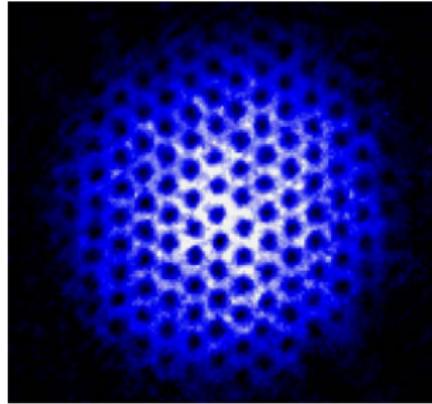
$$\mathbf{v} = \frac{\hbar}{m} \nabla \theta \quad \text{superfluid velocity} \quad \text{irrotational flow} \Rightarrow \nabla \times \vec{\mathbf{v}} = \vec{0}$$

velocity of the flow

Hydrodynamics and superfluidity

Hydrodynamics equations enable to describe

- ▶ The expansion of a quantum gas
- ▶ The low energy collective modes of a trapped gas (breathing, quadrupole, scissors modes...)
- ▶ Excitations: phonons, solitons, free particles...
- ▶ The formation of vortices in the presence of rotation



$$\oint \vec{v} \cdot d\vec{l} = \frac{2\pi}{m} \frac{v}{r} l h = \frac{2\pi}{2\pi r} = \frac{1}{r}$$

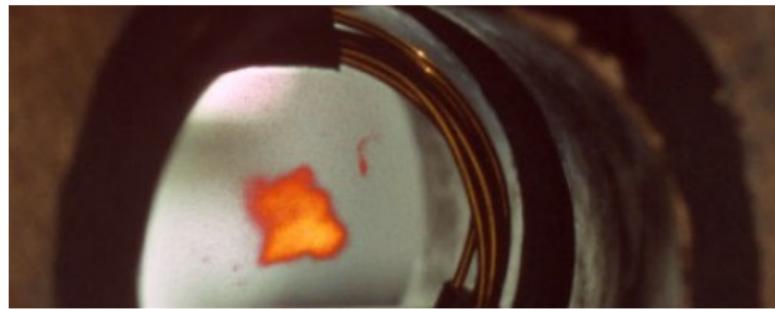
A signature of superfluidity: **vortex lattice** in a rotating condensate.



$$\frac{2\pi}{2\pi r} = \frac{1}{r}$$

Quantum gases in practice

Experimental methods



Experimental methods

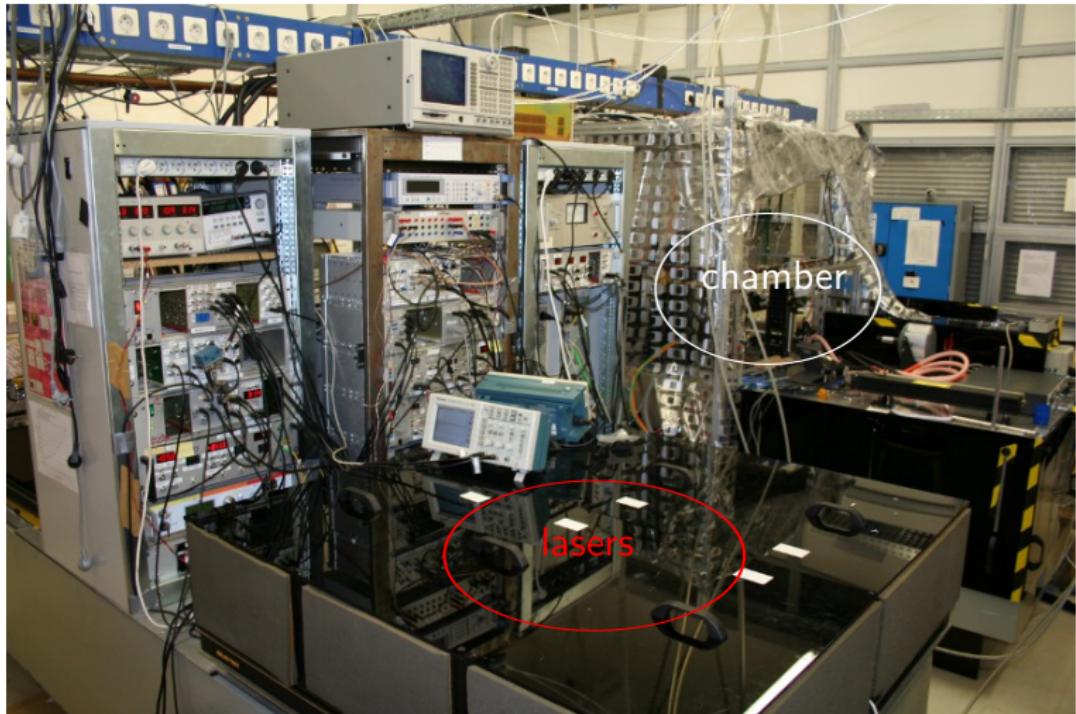
What does 'low temperature' mean?

cold: $T < 1 \text{ mK}$ **ultracold:** $T < 1 \mu\text{K}$

tools: lasers, ultrahigh vacuum, magnetic fields and RF fields, electronics, instrumentation...

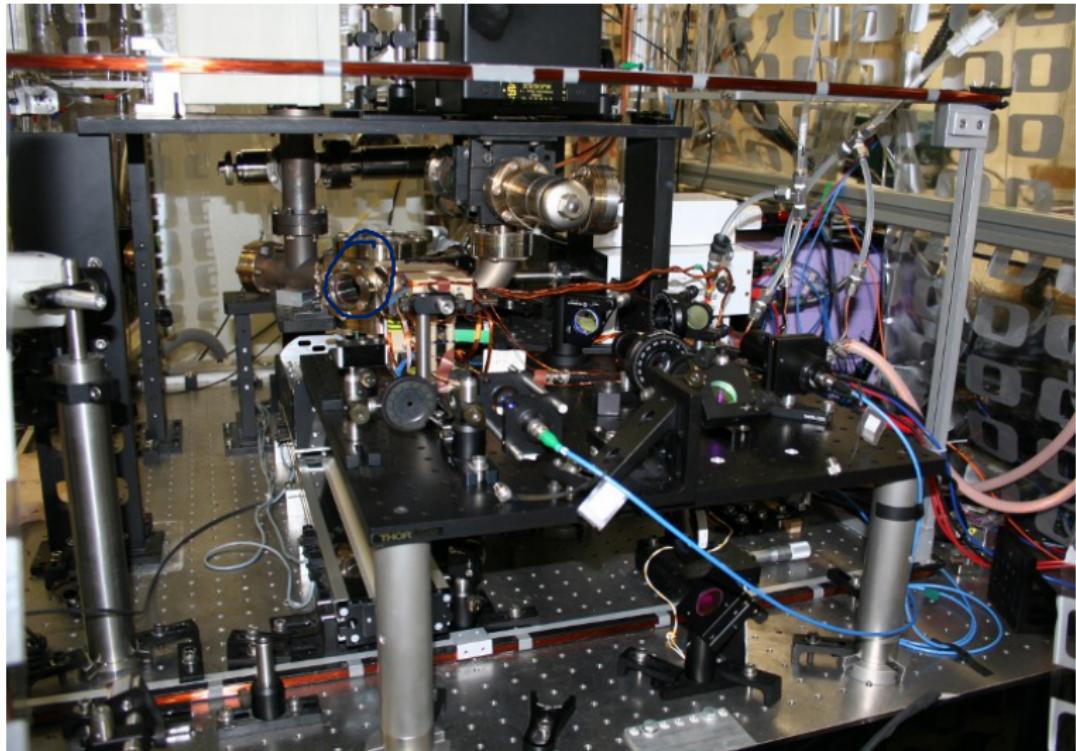

-11
 $\sim 10 \text{ mbar}$

A typical setup for Bose-Einstein condensation



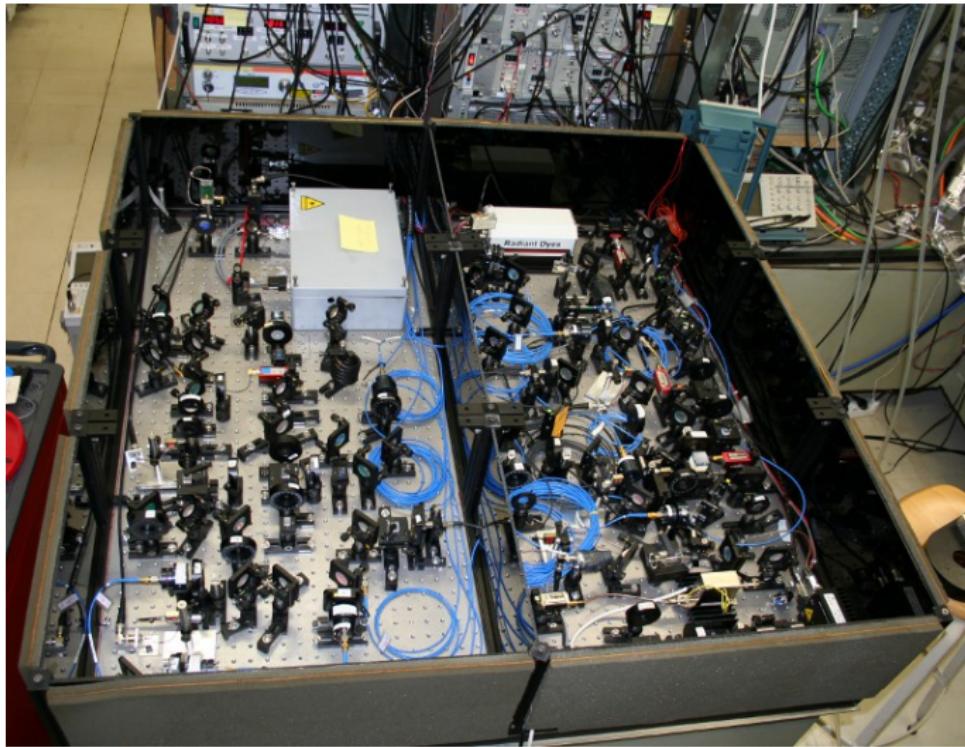
overall view (rubidium experiment, LPL)

A typical setup for Bose-Einstein condensation



vacuum chamber

A typical setup for Bose-Einstein condensation



laser sources

Reaching condensation

The traditional recipe

$$n\lambda^3 \sim 2.6$$

$$\frac{n}{\lambda^3} = \frac{n}{\sqrt{2\pi m k_B T}}$$

n: density

- A very high **vacuum** (10^{-11} mbar)

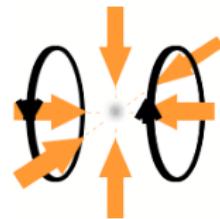
$$n\lambda^3 \sim 10^{-20}$$

- Precooling in a **magneto-optical trap** :

$$10^8\text{--}10^{10} \text{ atoms @ } 1\text{--}100 \mu\text{K} ; n\lambda^3 \sim 10^{-7}$$

$$\vec{F} = -\alpha \vec{v} - K \vec{r}$$

Doppler *resonant effect*



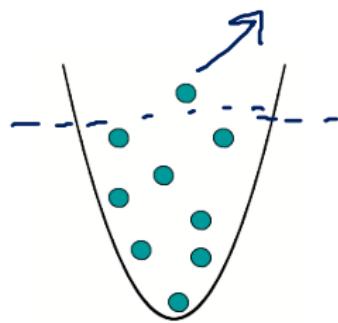
- transfer to a **conservative trap** (magnetic, optical, hybride...)
- evaporative** cooling down to T_C : $n\lambda^3 \sim 1$ *for detuned*
- resonant absorption (or fluorescence) **imaging** of the $10^4\text{--}10^6$ degenerate atoms @ 1–100 nK.



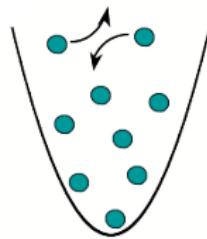
Evaporative cooling

... or how blowing on your coffee cools it down

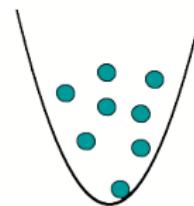
Lower the conservative trap depth: energy is redistributed via **elastic collisions**.



equilibrium at T_1



lower trap depth

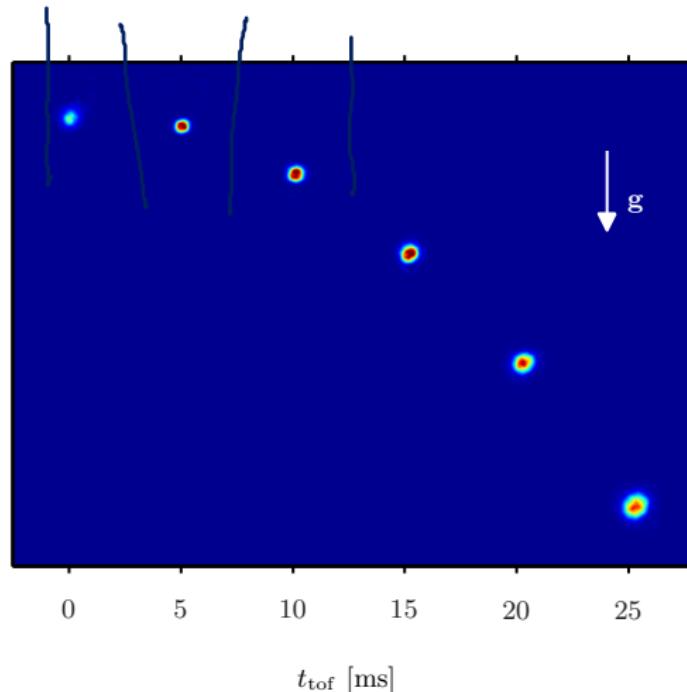


equilibrium at
 $T_2 < T_1$

Observation of the condensate expansion

Time-of-flight analysis

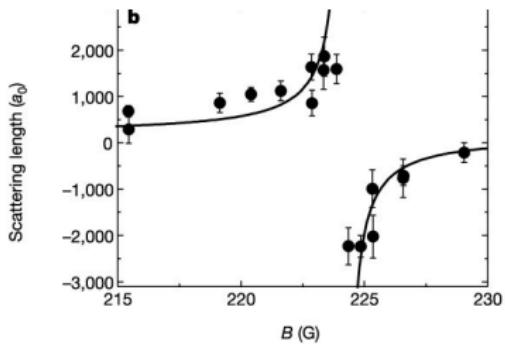
Time-of-flight: free expansion of a condensate due to the initial momentum width and the repulsive interactions.



expansion and free fall
of a rubidium
condensate (LPL 2011)

Quantum gases seen as tunable quantum systems

A widely tunable system



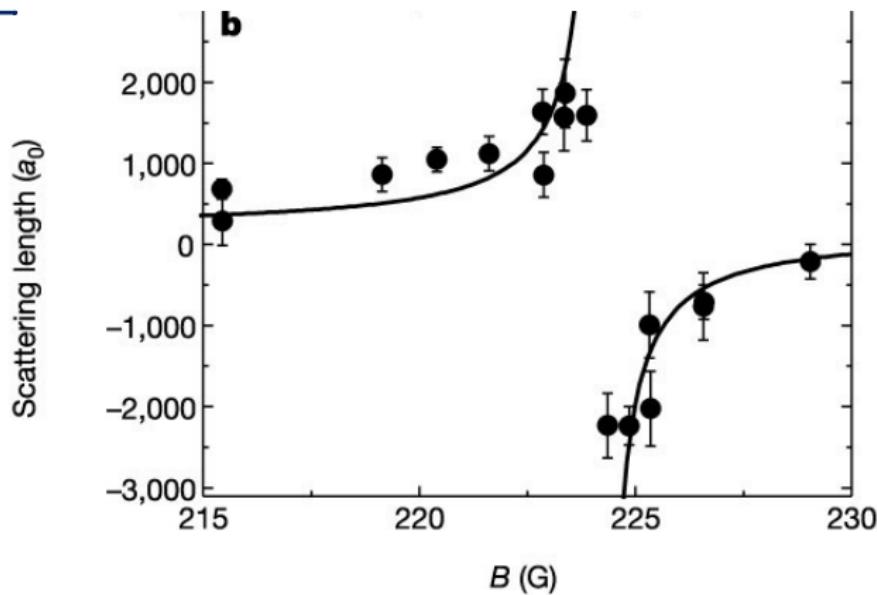
Quantum gases seen as tunable quantum systems

Which advantages?

- ▶ simple systems, well understood parameters
- ▶ extremely low temperatures accessible down to $\sim 1 \text{ nK}$
- ▶ bosons, fermions, mixtures have been trapped and cooled to degeneracy
- ▶ tunable interactions ; ; ; ; $g = \frac{4\pi\hbar^2 a}{m}$ a can be tuned
- ▶ large variety of trapping geometries (incl. low dimensions)
- ▶ effective magnetic field (with gauge field)
- ▶ large variety of direct or indirect probes
- ▶ natural analogy with **optics** (atom laser, quantum optics...)
- ▶ analogy with **condensed matter systems** (atoms playing the role of electrons)
- ▶ good candidate for Feynman's **quantum simulator**

Control of interactions

- ▶ s-wave collisions: modelled by a contact interaction $g\delta(r)$
- ▶ $g = \frac{4\pi\hbar^2 a}{M}$ a : **scattering length**
- ▶ tune the scattering length with a magnetic field using Feshbach resonances



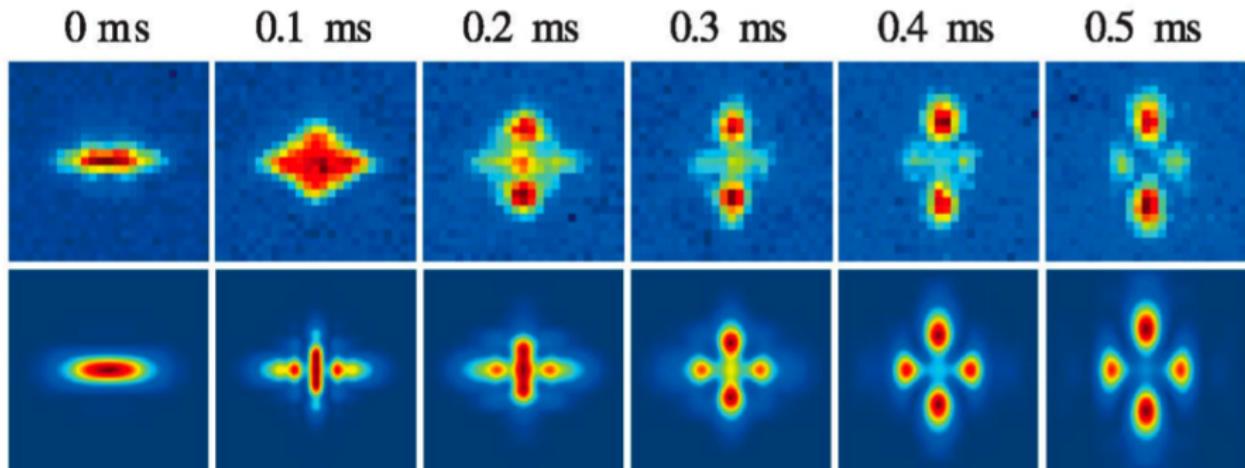
Control of interactions

Example: collapse of a dipolar gas

$$\begin{array}{c} 6e^- \\ \uparrow \uparrow \\ \mu = 6\mu_B \end{array}$$

spin $\frac{1}{2} \times 6$

Collapse of a chromium dipolar gas, obtained by setting $a = 0$.

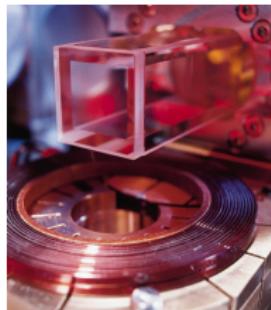


Pfau (Stuttgart), 2008



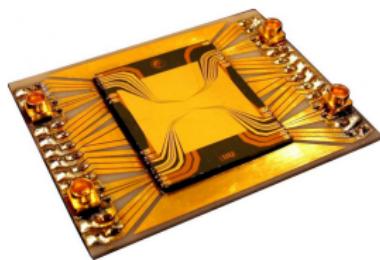
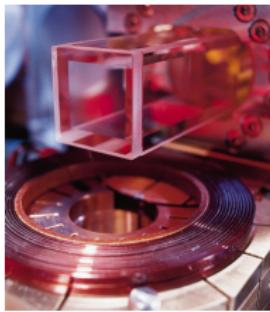
Gallery of conservative traps

- ▶ magnetic traps: from macroscopic coils...



Gallery of conservative traps

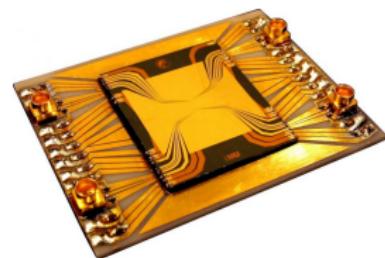
- ▶ magnetic traps: from macroscopic coils...



... to atom chips

Gallery of conservative traps

- ▶ magnetic traps: from macroscopic coils...

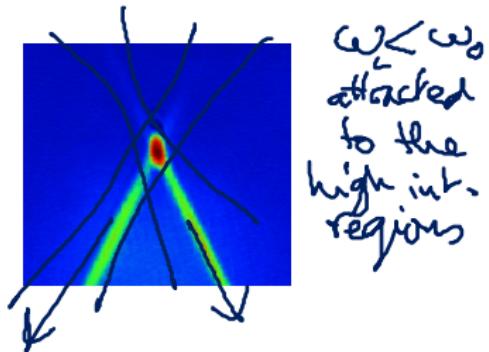


... to atom chips

- ▶ light traps

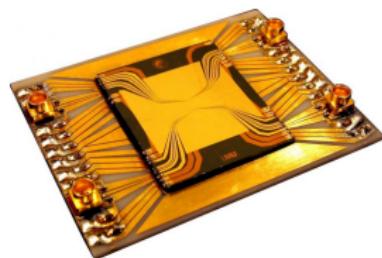
potential \propto intensity; repulsive or attractive depending on laser detuning with respect to atomic resonance

$\omega_L > \omega_0$ repelled from high int. regions



Gallery of conservative traps

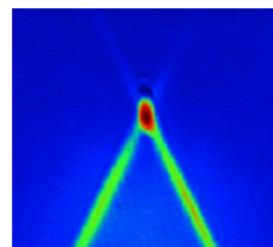
- ▶ magnetic traps: from macroscopic coils...



... to atom chips

- ▶ light traps

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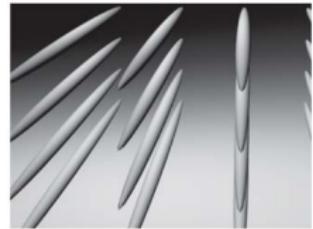
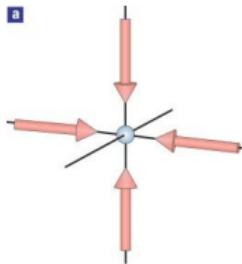


- ▶ adiabatic potentials (RF traps)

Various trapping geometry



- **harmonic traps** : 3D, 2D, 1D

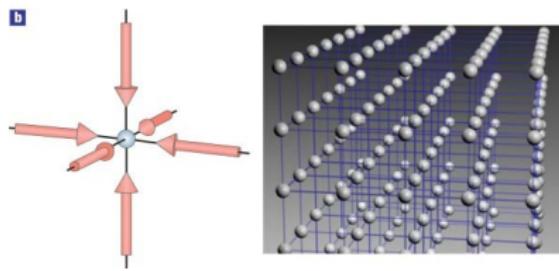


Ex: series of 1D tubes

Various trapping geometry

- ▶ harmonic traps : 3D, 2D, 1D
- ▶ **optical lattices**

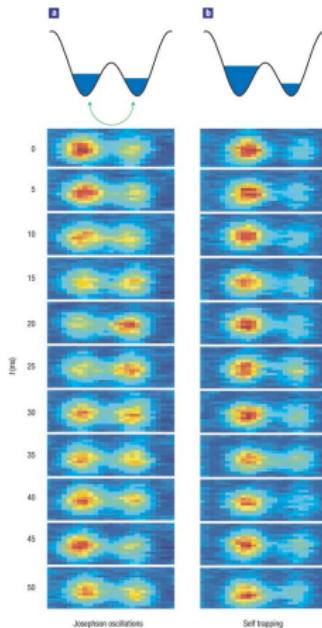
a 3D optical lattice is the analogue of the ion crystal lattice



Various trapping geometry

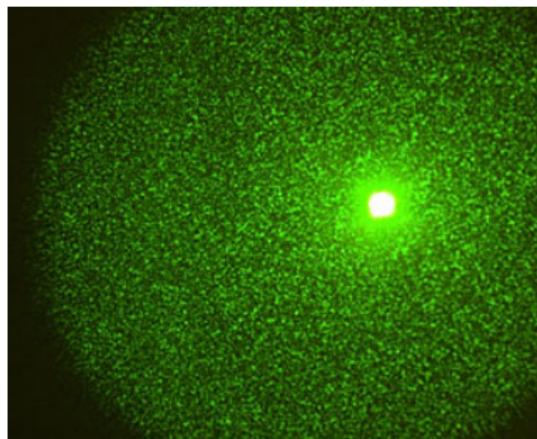


- ▶ harmonic traps : 3D, 2D, 1D
- ▶ optical lattices
- ▶ **double well:**
M. Oberthaler 2005
Josephson oscillations



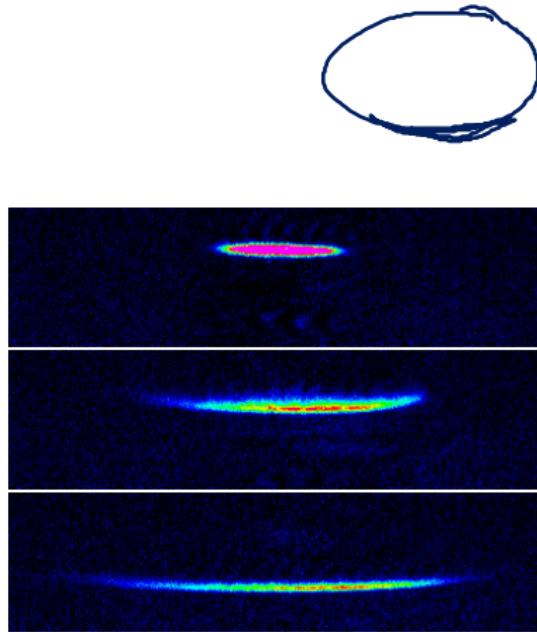
Various trapping geometry

- ▶ harmonic traps : 3D, 2D, 1D
- ▶ optical lattices
- ▶ double well
- ▶ **disordered potentials**



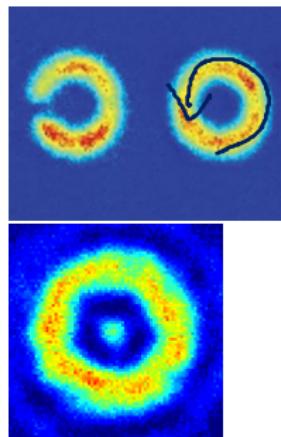
Various trapping geometry

- ▶ harmonic traps : 3D, 2D, 1D
- ▶ optical lattices
- ▶ double well
- ▶ disordered potentials
- ▶ curved traps: LPL



Various trapping geometry

- ▶ harmonic traps : 3D, 2D, 1D
- ▶ optical lattices
- ▶ double well
- ▶ disordered potentials
- ▶ curved traps
- ▶ **ring traps:**
NIST 2011 / LPL



$$\oint \vec{v} \cdot d\vec{l} = \frac{n\hbar}{M}$$

These potentials can be modified in a dynamical way.

Observables

- ▶ **Absorption imaging**
- ▶ Time of flight
- ▶ High resolution fluorescence
- ▶ Bragg spectroscopy
- ▶ RF spectroscopy
- ▶ measurement of local correlations
- ▶ ...

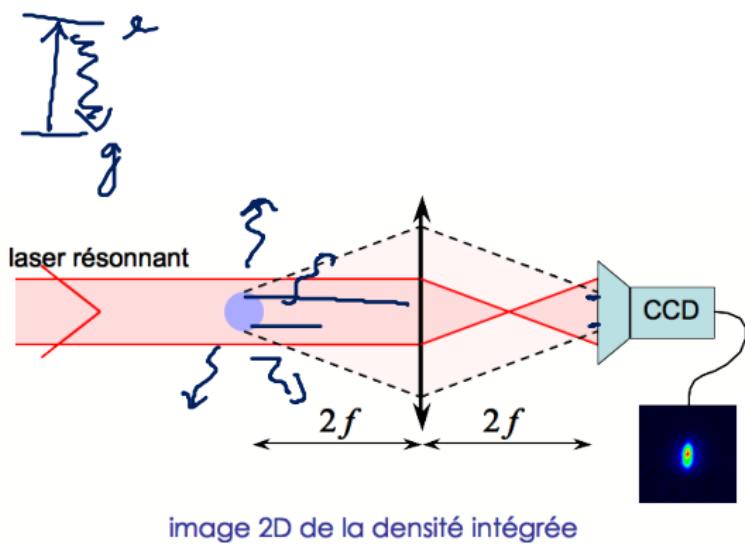


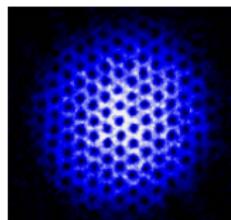
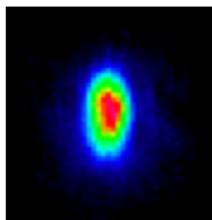
image 2D de la densité intégrée

Observables



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- ▶ **Time of flight**
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- ▶ Bragg spectroscopy
- ▶ RF spectroscopy
- ▶ measurement of local correlations
- ▶ ...

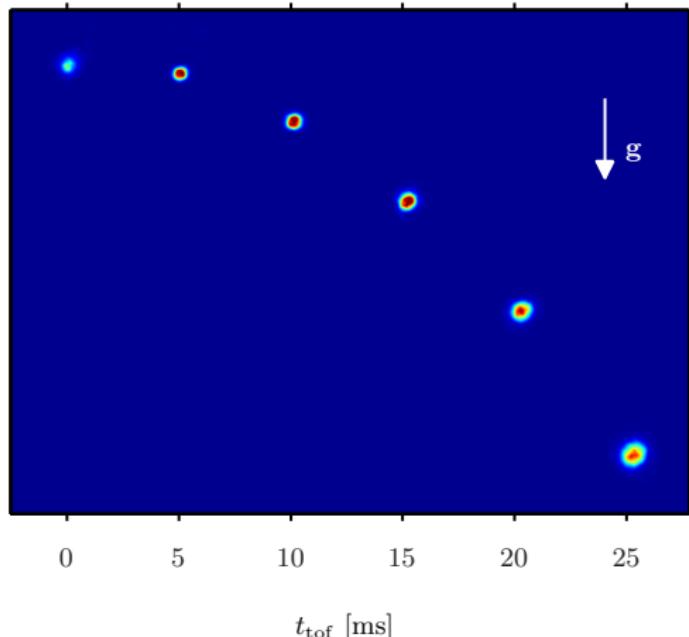
momentum distribution...
... or zoom on initial state



Observables

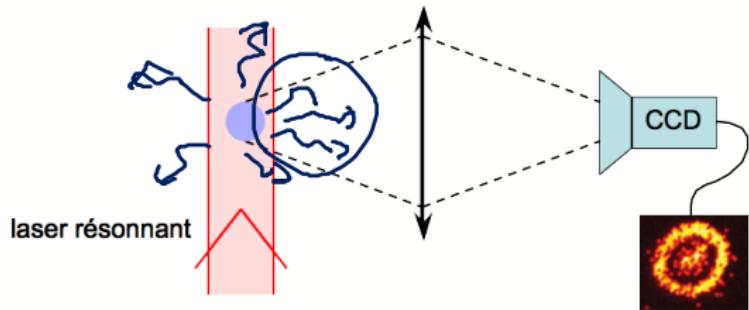
- ▶ Absorption imaging
- ▶ **Time of flight**
- ▶ High resolution fluorescence
- ▶ Bragg spectroscopy
- ▶ RF spectroscopy
- ▶ measurement of local correlations
- ▶ ...

expansion and free fall of a BEC (LPL)



Observables

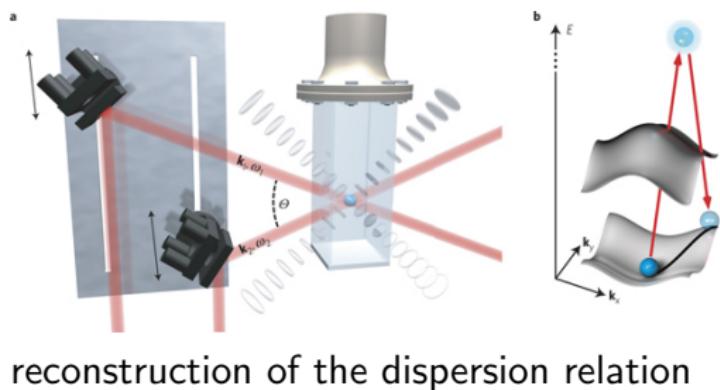
- ▶ Absorption imaging
- ▶ Time of flight
- ▶ **High resolution fluorescence**
- ▶ Bragg spectroscopy
- ▶ RF spectroscopy
- ▶ measurement of local correlations
- ▶ ...



S. Kuhr, Munich

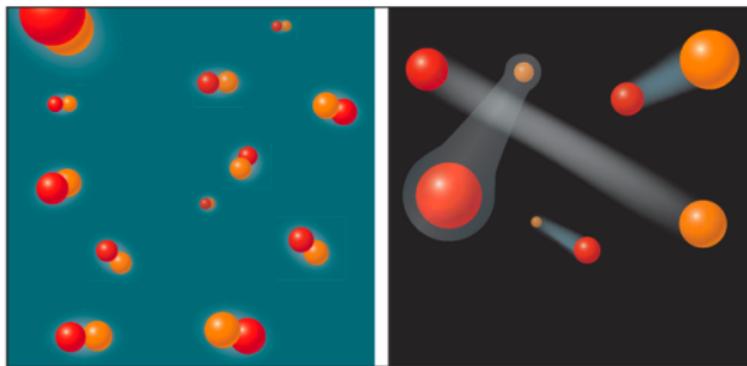
Observables

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- ▶ Time of flight
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- ▶ **Bragg spectroscopy**
- ▶ RF spectroscopy
- ▶ measurement of local correlations
- ▶ ...



Quantum gases as a model system

A model system for condensed matter



Tango or twist? In a magnetic field, atoms in different spin states can form molecules (left). Vary the field, and they might also form loose-knit Cooper pairs.

Science

A model system for condensed matter

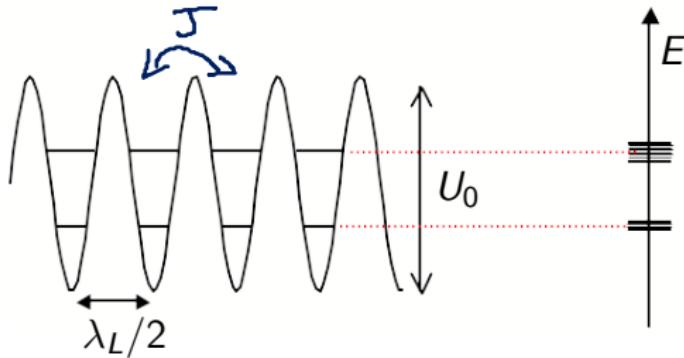
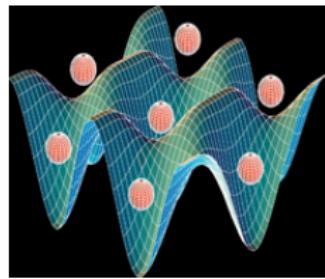
- ▶ Bloch oscillations
- ▶ metal – insulator Mott transition
- ▶ spins on lattice
- ▶ Josephson oscillations
- ▶ Anderson localization
- ▶ superfluidity of bosons or paired fermions (BCS)
- ▶ Abrikhov vortex lattice in a superfluid / a type-II superconductor
- ▶ low dimensional physics (2D: Berezinskii-Kosterlitz-Thouless transition, quantum Hall effect; 1D: Tonks-Girardeau gas, Luttinger liquid)
- ▶ ...

A model system for condensed matter

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Optical lattices

Optical lattices: far off resonance standing light waves.



Important parameters:

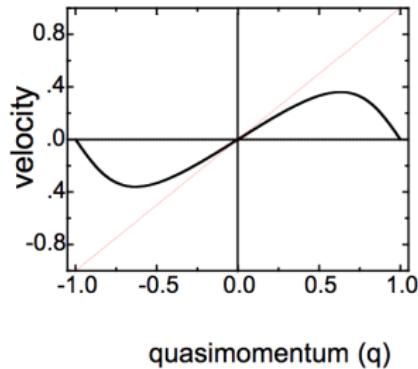
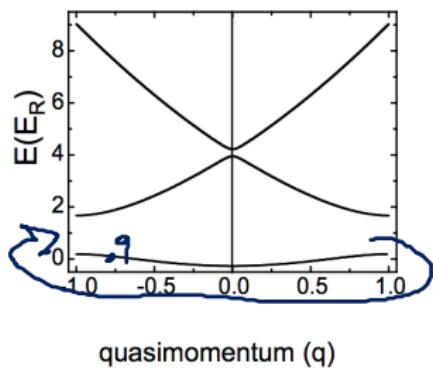
- ▶ period $\lambda_L/2$; 1st Brillouin zone $-\hbar k_L < q < \hbar k_L$
- ▶ band gap $\sim \hbar\omega_{\text{osc}} = 2\sqrt{U_0 E_{\text{rec}}}$ with $E_{\text{rec}} = \frac{\hbar^2 k_L^2}{2M}$
- ▶ bandwidth of fundamental band / tunnel coupling:

$$J \propto \delta E \propto e^{-2\sqrt{U_0/E_{\text{rec}}}}$$
 can be made very small
- ▶ effective mass in the fundamental band:

$$M_{\text{eff}} \propto 1/\delta E$$
 can be made very large

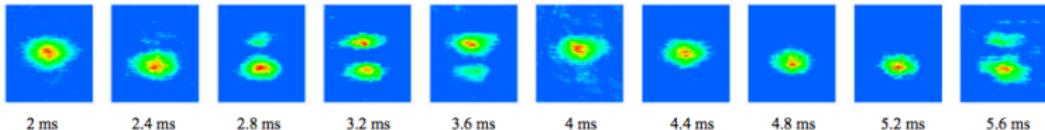
Bloch oscillations in the fundamental band

Structure of the fundamental band:



The velocity oscillates in the presence of a constant external force:

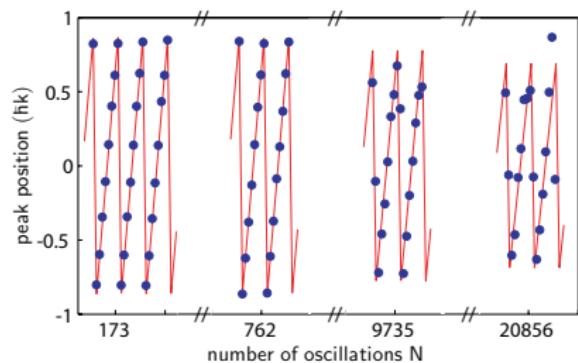
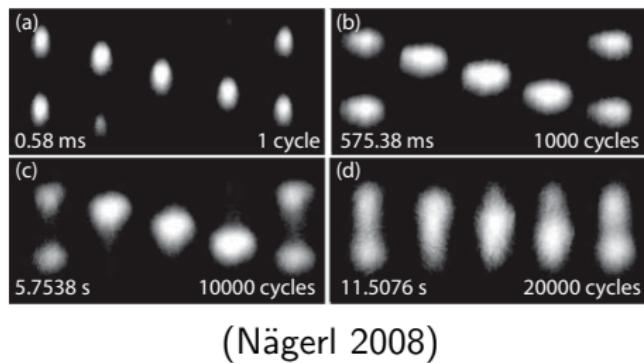
$$\dot{q} = \frac{1}{\hbar} F \Rightarrow q(t) = F \frac{t}{\hbar} \quad \dot{v} = v(t) = \frac{1}{\hbar} \frac{\partial E}{\partial q}(q(t))$$



Bloch oscillations with a non interacting BEC

With a cesium BEC – Feshbach resonance such that $a_s = 0$.

$$\text{Bloch period: } \tau = \frac{2\hbar}{Mg\lambda_L}$$



Up to 20000 oscillations!

⇒ use Bloch oscillations for measuring g or h/M .

Mott transition

A quantum phase transition at $T = 0$

Bose-Hubbard Hamiltonian, $\hat{n}_i = \hat{b}_i^\dagger \hat{b}_i$ number of particles per site

$$\therefore \text{3U}$$

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_j^\dagger \hat{b}_i + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

$$\therefore \text{4U}$$



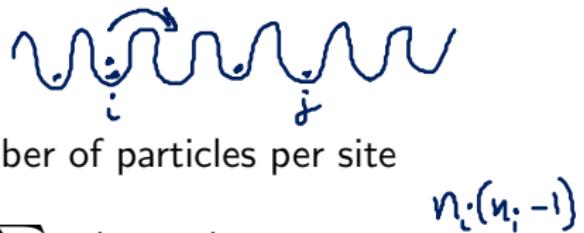
J : tunnel coupling

U on-site interaction energy

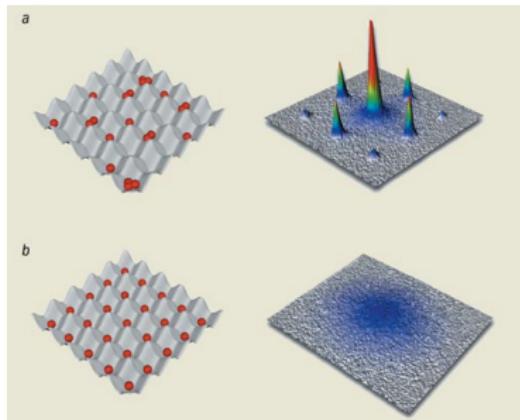
$U < J$ superfluid: coherent state $|\phi\rangle$

(delocalized wavefunction)

$U > J$ Mott insulator: Fock state $|N\rangle$



$$\frac{n_i(n_i-1)}{2}$$



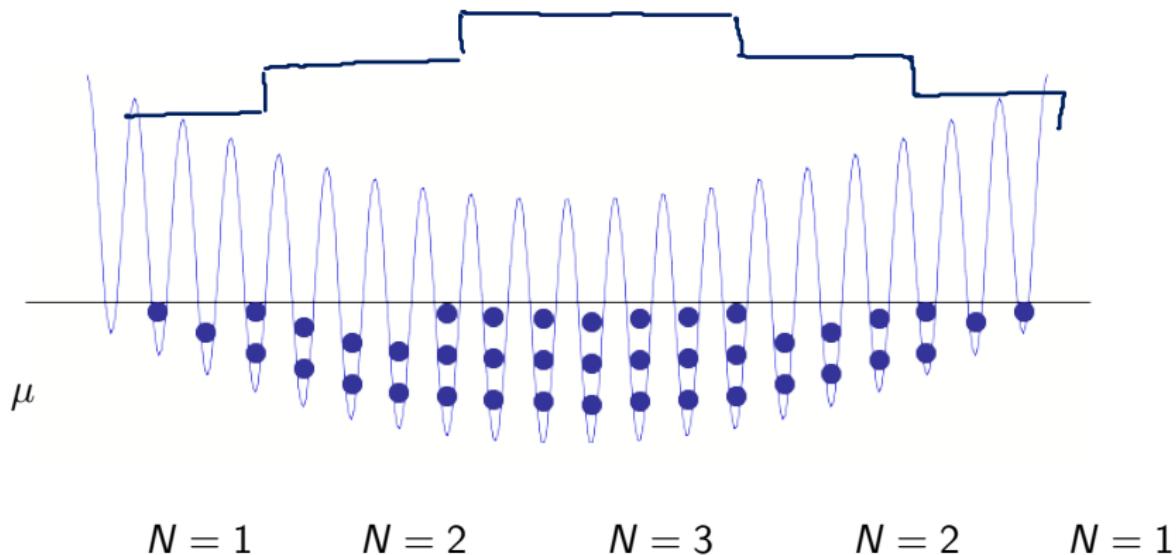
Greiner / Bloch / Esslinger 2002

The Mott insulating state is strongly correlated. Application:
entanglement for quantum information.

The Mott insulating state in a trap

Inhomogeneous gas

The Mott state is organised in concentric shells with different filling.



The density is expected to present steps (wedding cake).

Mott transition revealed by single atom imaging

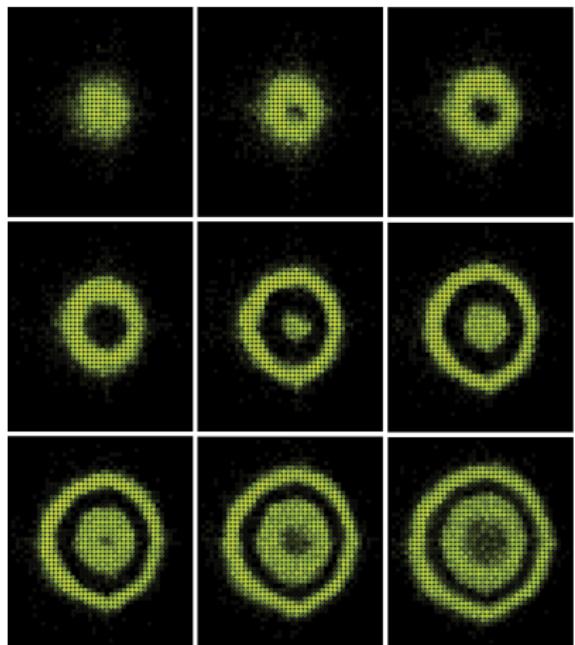
High resolution fluorescence imaging:

the layers with $N = 1, 2, 3$ atoms per site are identified by single-atom imaging.

Site with an **odd number of atoms** appear as bright.

M. Greiner 2011

S. Kuhr et I. Bloch 2011

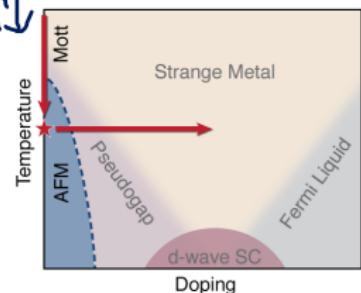


Exploring the phase diagram of Fermi-Hubbard model

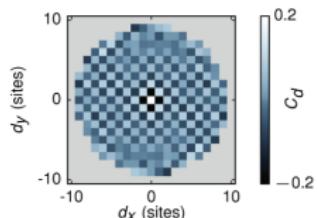
Fermions in two spin states – M. Greiner 2016



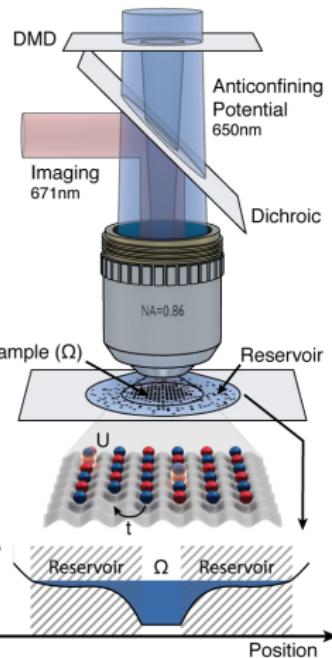
The Hubbard Model



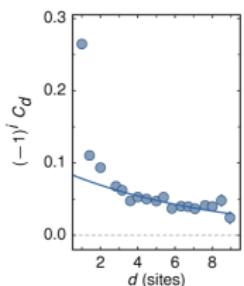
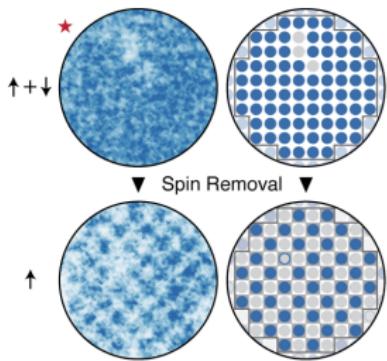
Spin Correlation Function



Entropy Redistribution



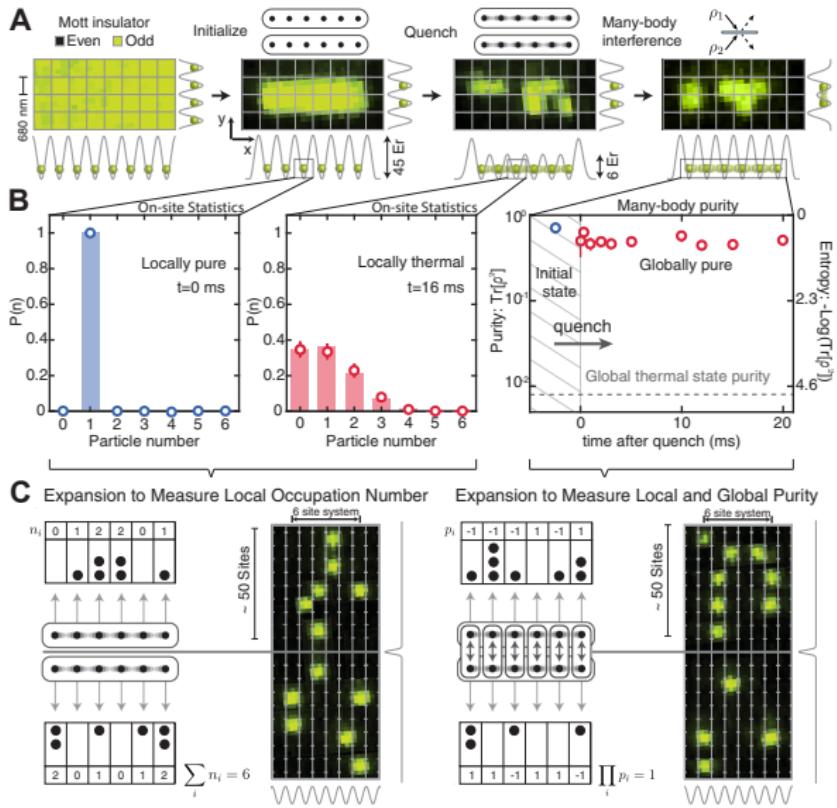
Long-range Antiferromagnet



Quantum simulation with a few atoms

Thermalization of small ensembles after a quench

M. Greiner 2016

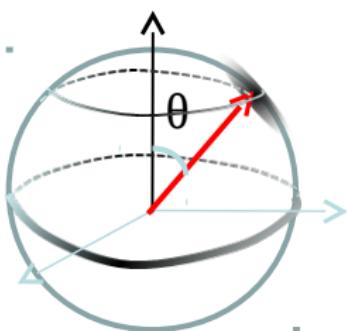


Out of equilibrium dynamics of large spins on a lattice

Chromium atom: $S = 3 \Rightarrow 7$ spin states

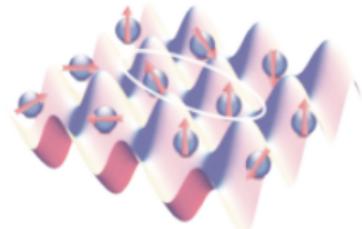
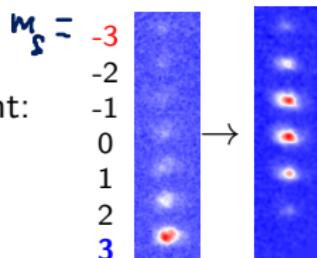
Initial state: Mott insulator, filling 1, spin up ($m_S = 3$)

$t = 0$: sudden spin rotation



$$\text{DDI} \propto \frac{1}{R^3}$$

Detection: Stern Gerlach experiment:
measure of spin populations



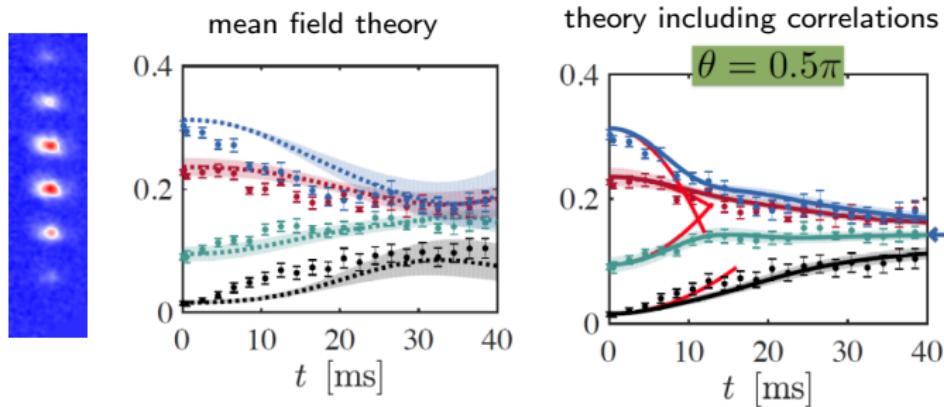
Parameters of the dynamics:

- ▶ Tunneling between sites
- ▶ van der Waals interactions
- ▶ Dipolar interactions

[Bruno Laburthe group, LPL Paris 13 / Paris Nord]

Out of equilibrium dynamics of large spins on a lattice

Mean field description **fails**. The spin dynamics reveals **particle correlations**
 \Rightarrow **many-body physics** at play.



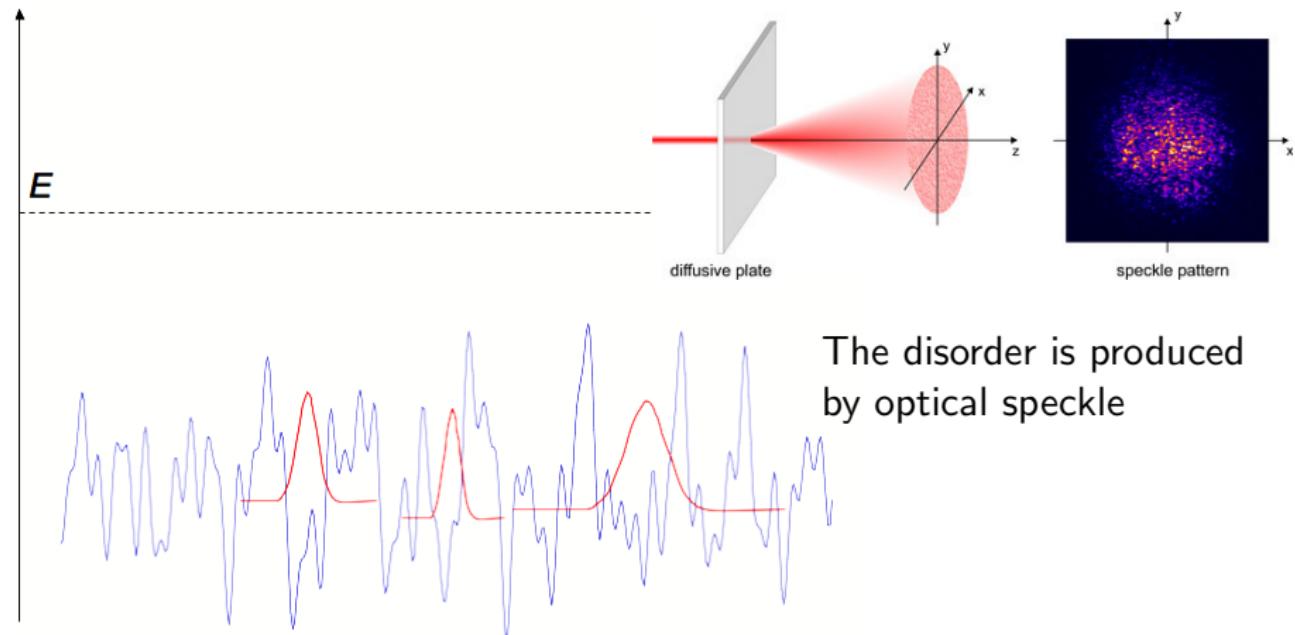
Full theory possible only at **short timescales** \Rightarrow long time evolution given by the quantum simulator
 Challenge in this system: **measure entanglement**.



[Bruno Laburthe group, LPL Paris 13 / Paris Nord]

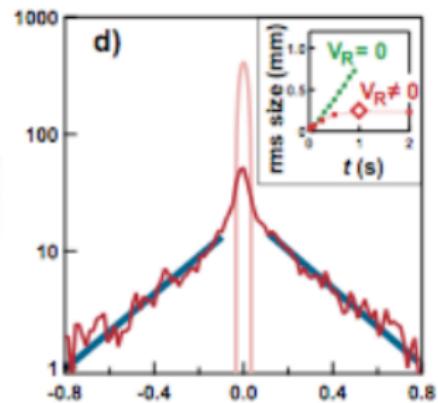
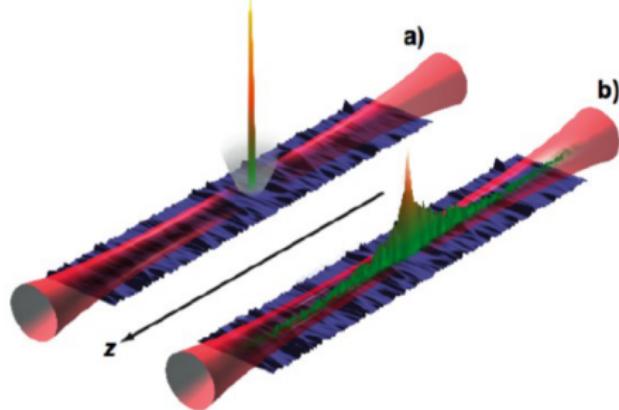
Anderson localization

Non trivial localization effect due to destructive interferences ($E > V_D$)



Anderson localization in 1D

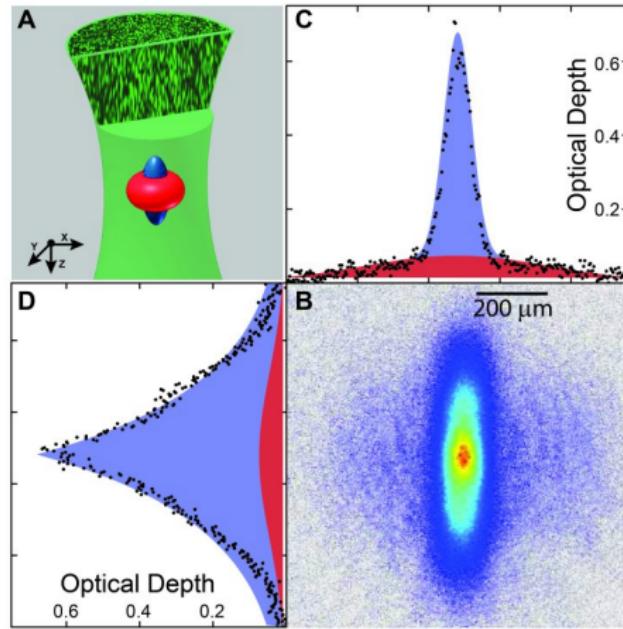
Exponential localization of the atom velocity distribution.



V. Josse / P. Bouyer / A. Aspect 2008
 C. Fort / M. Inguscio 2008

Anderson localization in 3D

Observed in 3D in 2011 by B. DeMarco (Illinois) & Josse / Aspect

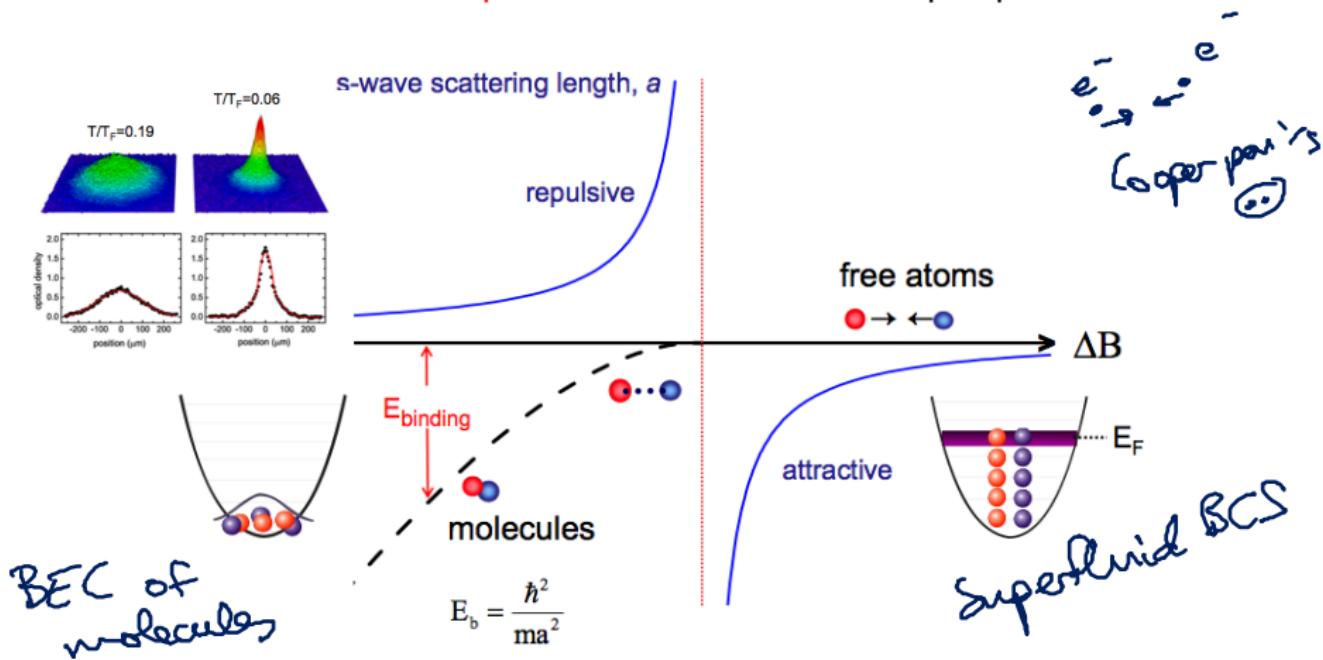


BCS state in degenerate fermions

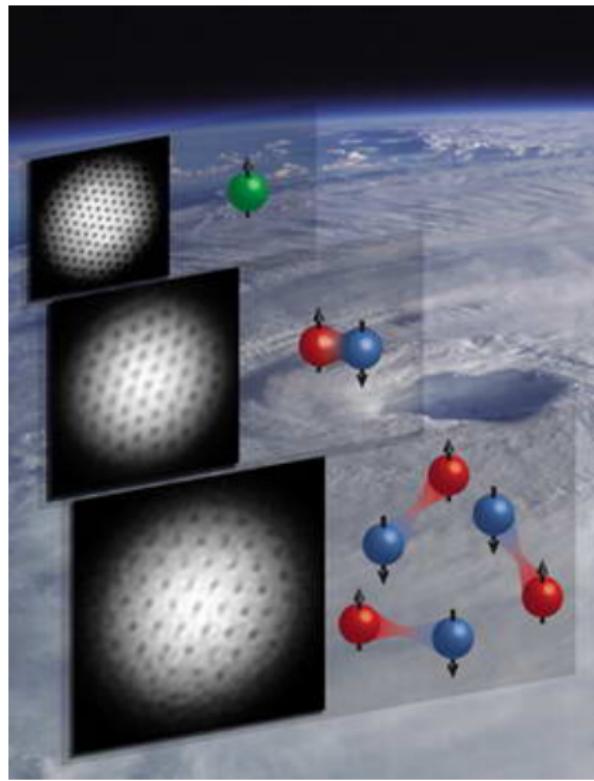
Pairing of fermions with opposite spin

Repulsive interactions: BEC of bosonic molecules

Attractive interactions: superfluid BCS state of Cooper pairs



Superfluidity of fermion pairs in the BEC-BCS crossover



atomic BEC

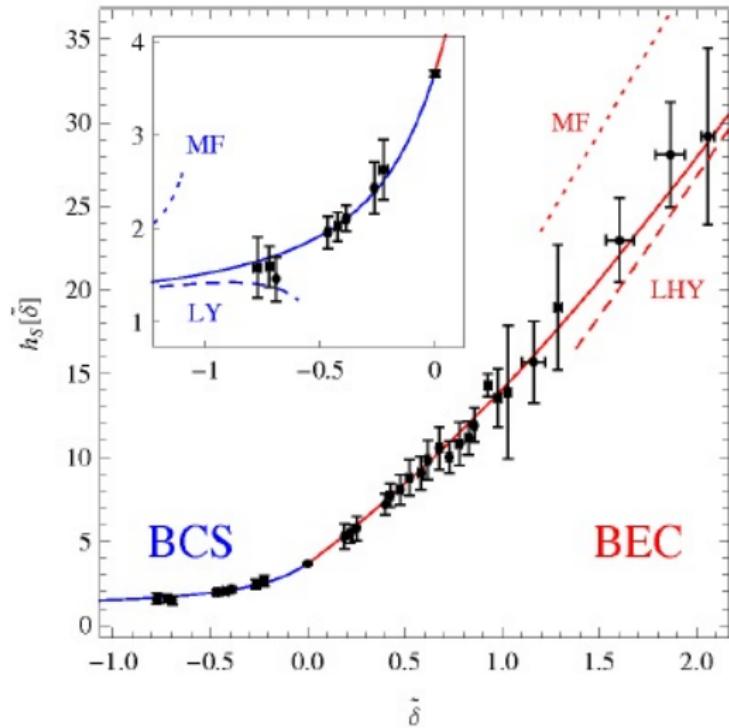
molecular BEC

superfluid of Cooper pairs

W. Ketterle 2006

Equation of state of the interacting Fermi gas

$$N \curvearrowright E_F$$



$$\mathcal{E} = (\) E_F$$

Unitary Fermi gas:

Limit $a \rightarrow +\infty$

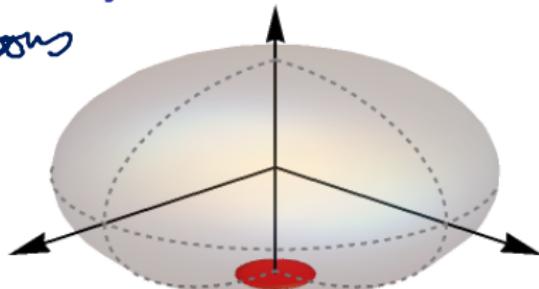
$$\text{and } \sigma = \frac{4\pi}{k^2}$$

Many-body physics of strongly interacting particles, difficult to **compute**, easier to **measure**

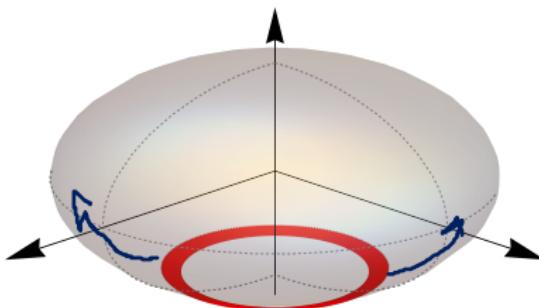
F. Chevy / C. Salomon 2010

Superfluidity in the fast rotation regime

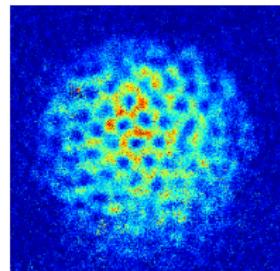
of bosons



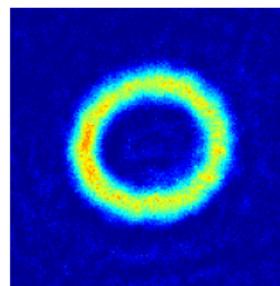
$$\Omega = 2\pi \times 24$$



$$\Omega = 2\pi \times 35.8 \text{ Hz}$$



vortex lattice

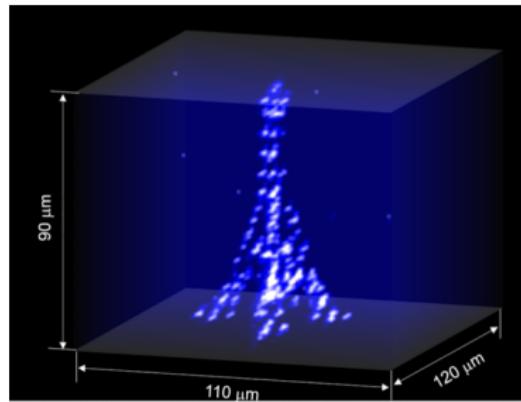


supersonic flow at
Mach 18

Y. Guo et al., PRL **124**, 025301 (2020) (BEC group@LPL)

Rydberg atoms in optical tweezers

Individually controlled atoms



Properties of Rydberg states

Scaling with the principal quantum number n



$$n \rightarrow 25 \dots \infty$$



- strong dipole moment: 10^{12} larger than that of magnetic atoms
- large size
- long life time: very little overlap with ground
- $1/R^3$ interaction ...

$$E_n = -\frac{Ry}{(n - \delta_{l,j})^2} \quad \text{Ry: Rydberg energy}$$

$\sim \frac{1}{n^2}$

$\delta_{l,j}$: quantum defect

$$E_{nl} - E_n \sim \frac{1}{n^3}$$

d scales as $n^2 q_0$

$$\text{Rydberg} \rightarrow \text{ground state}: R_{gs} \propto \frac{1}{n^3}$$

$$n \sim 50 \quad \Delta E \sim 50 \text{ GHz}$$

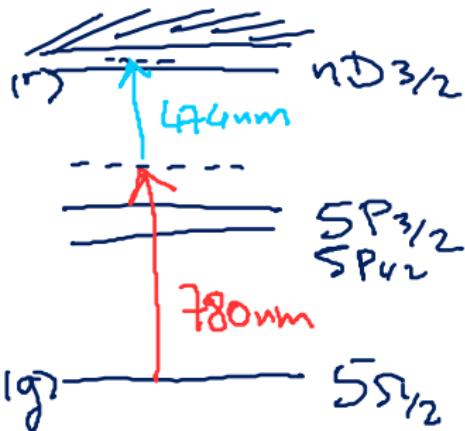
$$\text{lifetime} \sim \frac{n!}{n} \frac{\hbar}{\omega} \quad \Gamma \propto d^2 \omega^3$$

$$\propto \frac{1}{n^5} \text{ very small}$$

$$R \sim 200 \mu\text{m}$$

Properties of Rydberg states

State preparation: two-photon transition



Rb

2 photon - excitation

$|g\rangle \rightarrow |e\rangle$

Dipole-dipole interactions in Rydberg states

Scaling

$$V_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\vec{d}_1 \cdot \vec{d}_2 - 3(\vec{d}_1 \cdot \hat{n})(\vec{d}_2 \cdot \hat{n})}{R^3}$$

\vec{d} is an odd operator (spatial sym.)



$$\langle \alpha | \vec{d}^\dagger | \alpha \rangle = 0 \quad \langle \alpha \beta | V_{dd} | \alpha \beta \rangle = 0$$

Second order :

$$\Delta E_{dd} = \sum_{\beta, \gamma} \frac{\langle \alpha \alpha | V_{dd} | \beta \gamma \rangle \times \langle \beta \gamma | V_{dd} | \alpha \alpha \rangle}{E_\alpha + E_\beta - (E_\beta + E_\gamma)} \frac{k_{dd} |d_\beta| |d_\gamma|}{R^6}$$

$$= \frac{C_6}{R^6} \propto n^3 \cdot (n^2)^4 = n^{10}$$

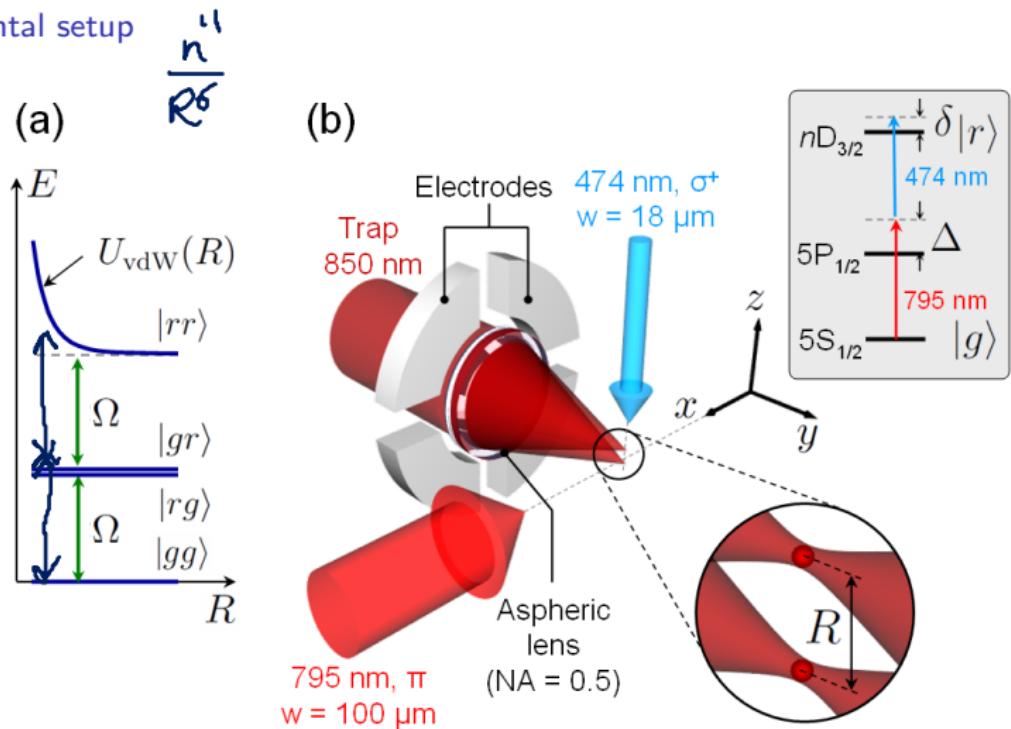
van der Waals interaction

$$\frac{\Delta E_{dd}}{h} \sim 10 \text{ MHz}$$

for $R \approx 10 \mu\text{m}$

Dipole-dipole interactions in Rydberg states

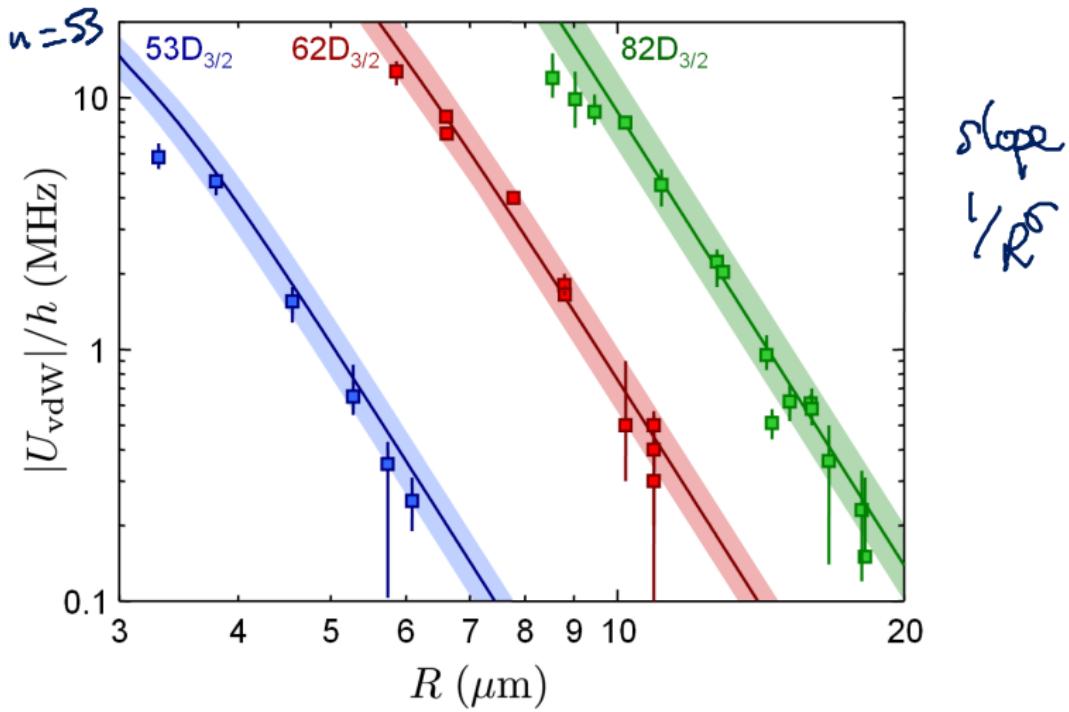
Experimental setup



[Béguin et al., PRL **110**, 263201 (2013)]

Dipole-dipole interactions in Rydberg states

C_6/R^6 dipole-dipole interaction



[Béguin et al., PRL **110**, 263201 (2013)]

Dipole-dipole interactions in Rydberg states $\sigma_+^1 \sigma_-^2$

Resonant DDI: spin-exchange coupling

$$\langle \alpha\beta | V_{dd} | \beta\alpha \rangle \propto \frac{C_3}{R^3}$$

$$\{\alpha\beta, \beta\alpha\}$$

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}} (|\alpha\beta\rangle \pm |\beta\alpha\rangle)$$

$$H = \frac{C_3}{R^3} (\sigma_+^1 \sigma_-^2 + \sigma_-^1 \sigma_+^2)$$

$$C_3 \propto n^4 \propto d^2$$

spin exchange interaction

$\longrightarrow |\beta\rangle$

$\longrightarrow |\alpha\rangle$

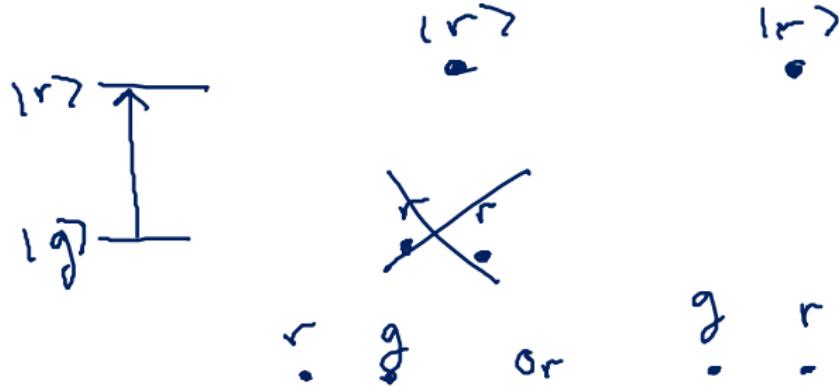
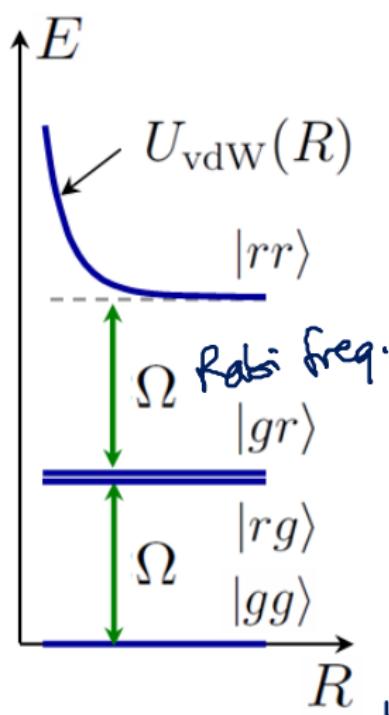
Försker resonance (with electric field)



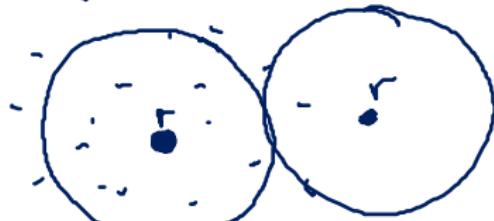
→ way to tune DDI between Rydgs

Dipole-dipole interactions in Rydberg states

Rydberg blockade



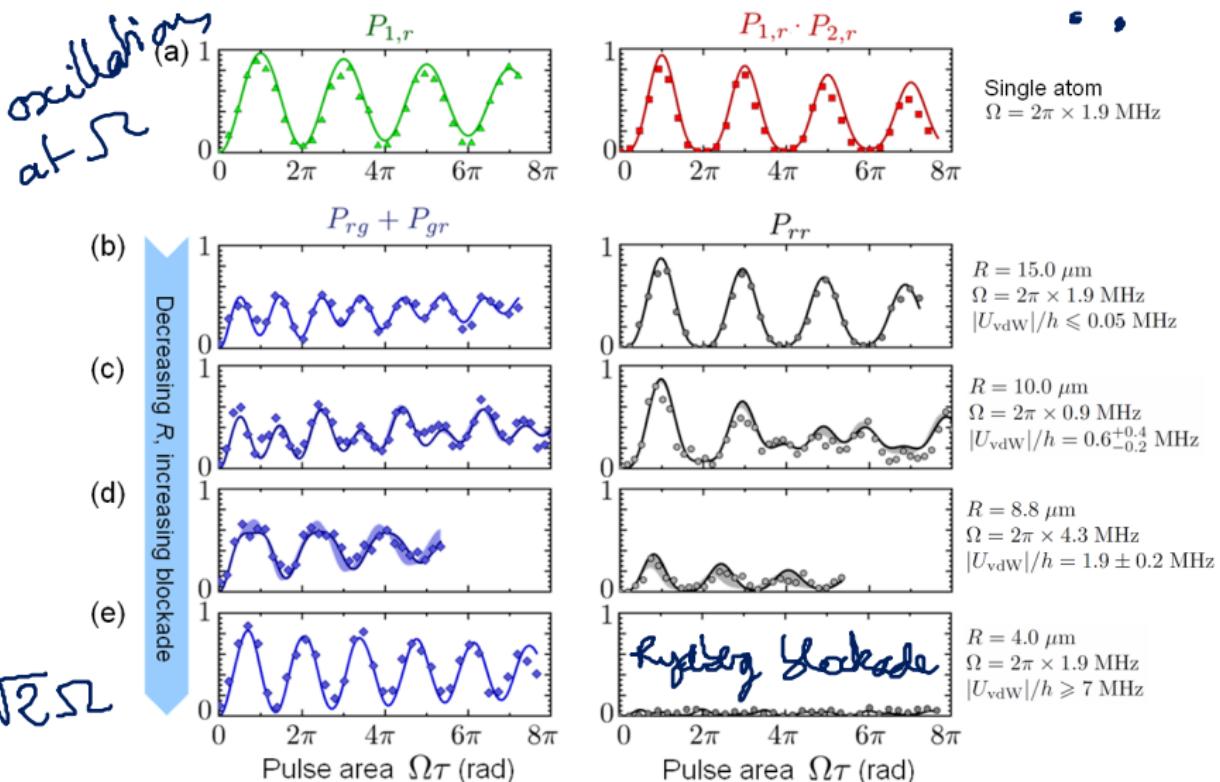
Rydberg blockade



$$|gg\rangle \leftrightarrow |4_{12}\rangle = \frac{1}{\sqrt{2}}(|rg\rangle + |gr\rangle)$$

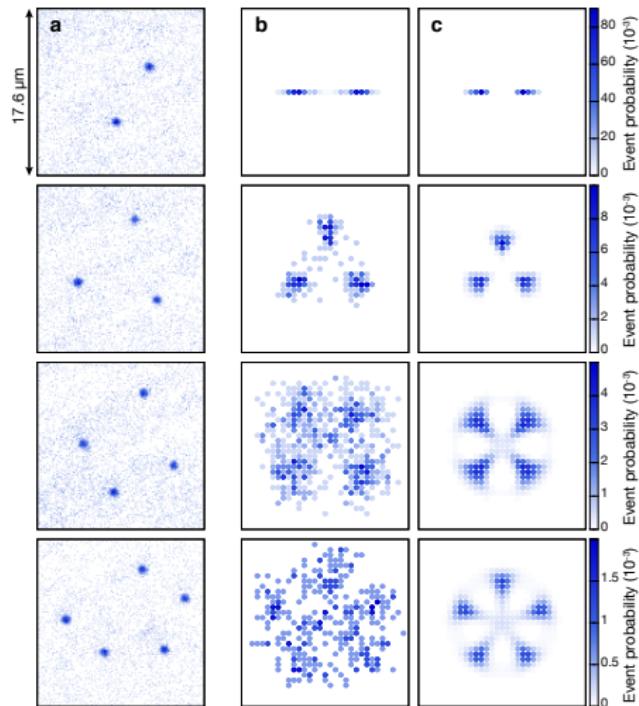
Dipole-dipole interactions in Rydberg states

Rydberg blockade [Béguin et al., PRL **110**, 263201 (2013)]

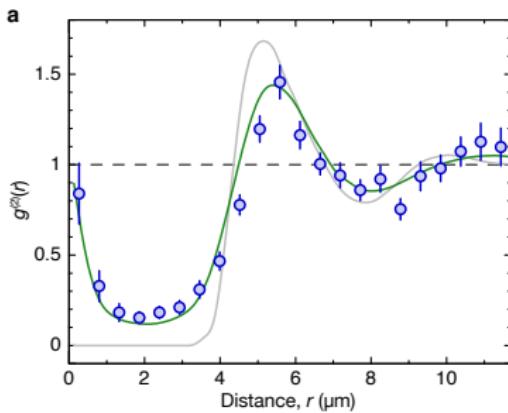


Correlations between Rydberg atoms

Rydberg patterns



- ▶ Rydberg excitation of atoms in a Mott state
- ▶ Removal of ground state atoms
- ▶ Single atom detection



[P. Schauss et al., Nature **491**, 87 (2012)]

Many-body physics with Rydberg atoms

Spin Hamiltonian simulation

Heisenberg model $H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$

Ising model $H = \sum_{ij} J_{ij} \hat{S}_i^z \hat{S}_j^z$

XY model $H = \sum_{ij} J_{ij} S_i^+ S_j^- + h.c.$

+ external field $B \sum_i S_i^z$ or $x^0 \wedge y^0 \dots$
 $(r \times r) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Use $|r\rangle, |g\rangle$ as effective spin state.

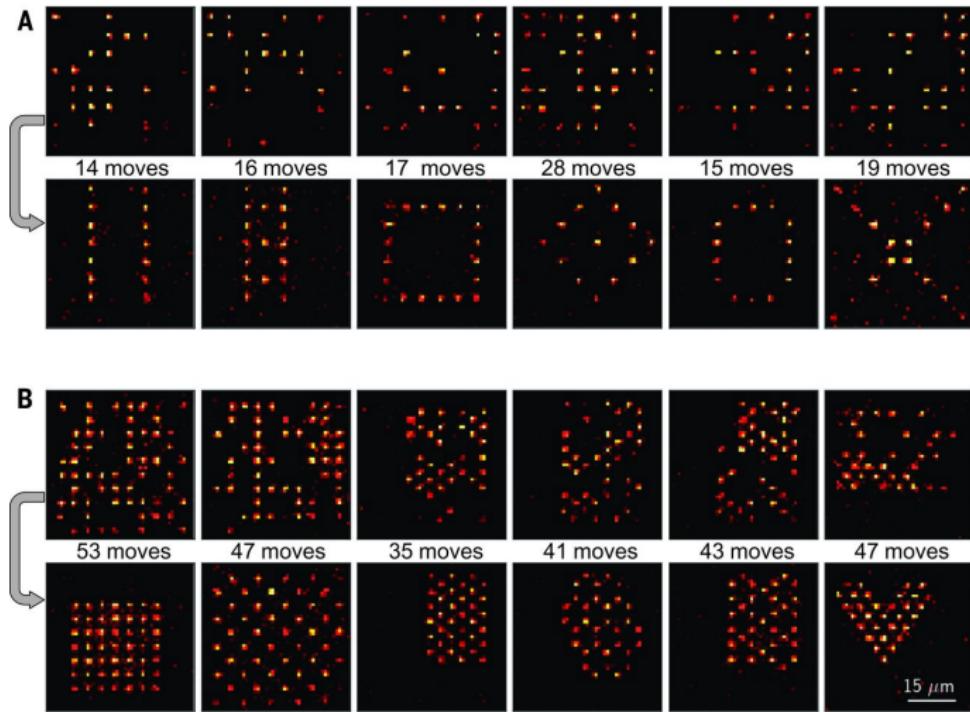
$|g\rangle \langle r| = \sigma_-$ or σ_z operator $= \frac{1}{2} + \frac{1}{2}\sigma_3$

$\hat{H}_{AL} = \hbar \omega \sum_i |\langle g_i | \langle r_i | + | r_i | g_i \rangle| = \hbar \omega \sum_i \sigma_x^i$

vdW: $f_{int} = \sum_{ij} \frac{C_6}{(r_i - r_j)^6} |\langle r_i | \langle r_j | + | r_j | r_i \rangle| = \sum_{ij} \frac{1}{4} (1 + \sigma_x^i \sigma_x^j + \sigma_z^i \sigma_z^j)$

Many-body physics with Rydberg atoms

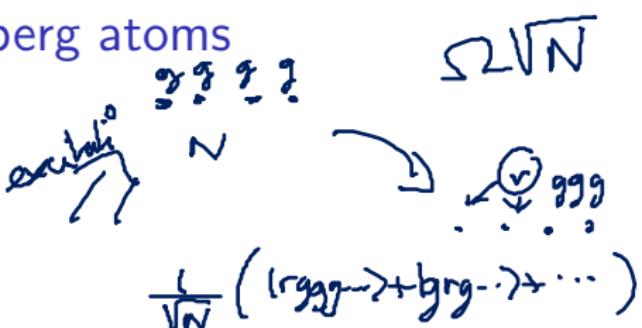
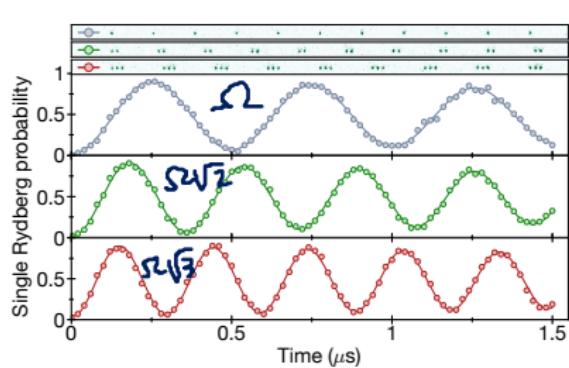
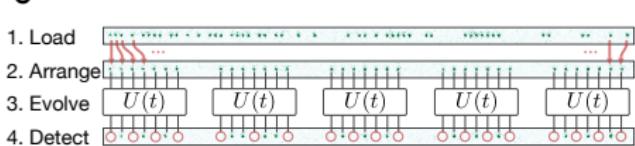
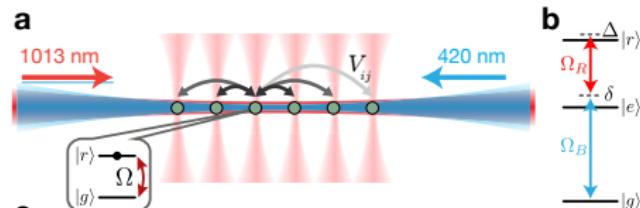
Ordering single trapped Rydberg atoms



[D. Barredo et al., Science 354, 1021 (2016)]

Many-body physics with Rydberg atoms

N -atom oscillations

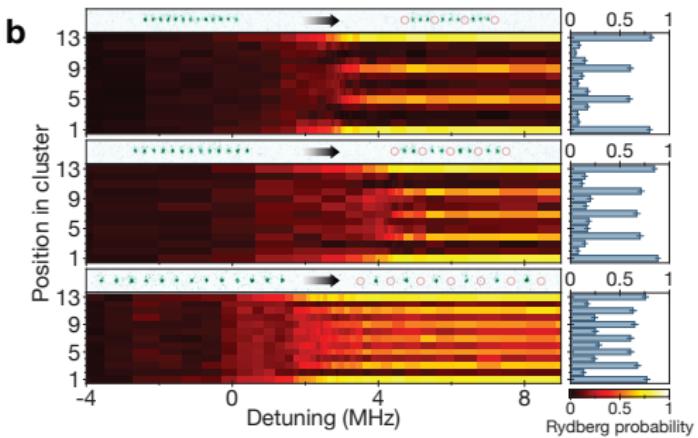
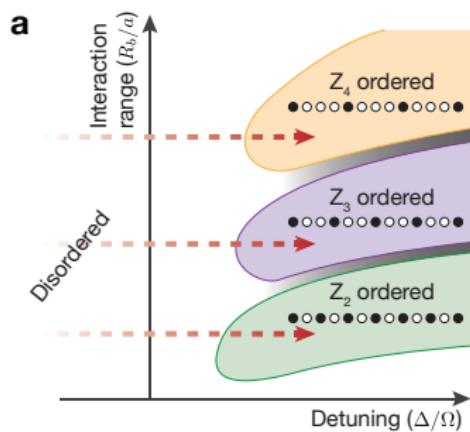


- ▶ array of single atoms in tweezers with stochastic filling
- ▶ rearrangement of filled sites
- ▶ collective excitations of one atom among 1 - 2 - 3

[H. Bernien et al., Nature 551, 579 (2017)]

Many-body physics with Rydberg atoms

Excitation patterns for 13 atoms



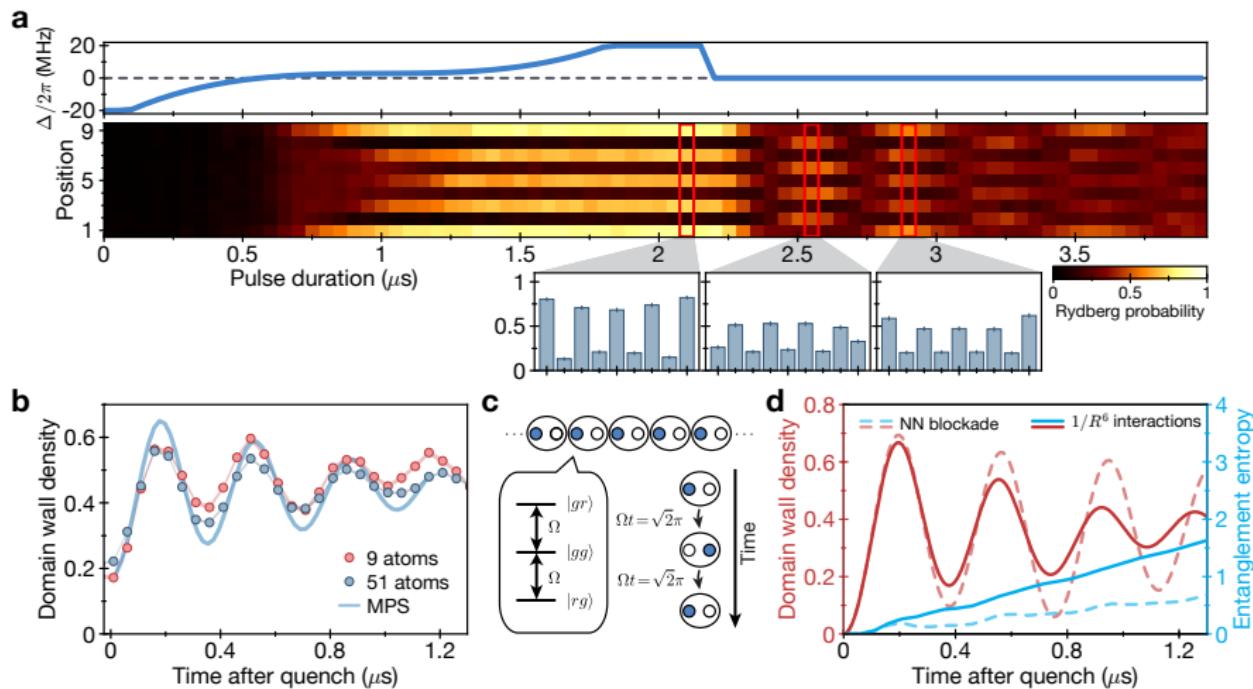
$$\hat{H} = \sum_i \frac{\hbar\Omega_i}{2} \hat{\sigma}_x^i - \sum_i \hbar\Delta_i \hat{n}_i + \sum_{i < j} V_{ij} \hat{n}_i \hat{n}_j$$

[H. Bernien et al., Nature 551, 579 (2017)]

$$\hat{n}_i = \frac{1}{2} (\hat{\sigma}_z^i + 1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Many-body physics with Rydberg atoms

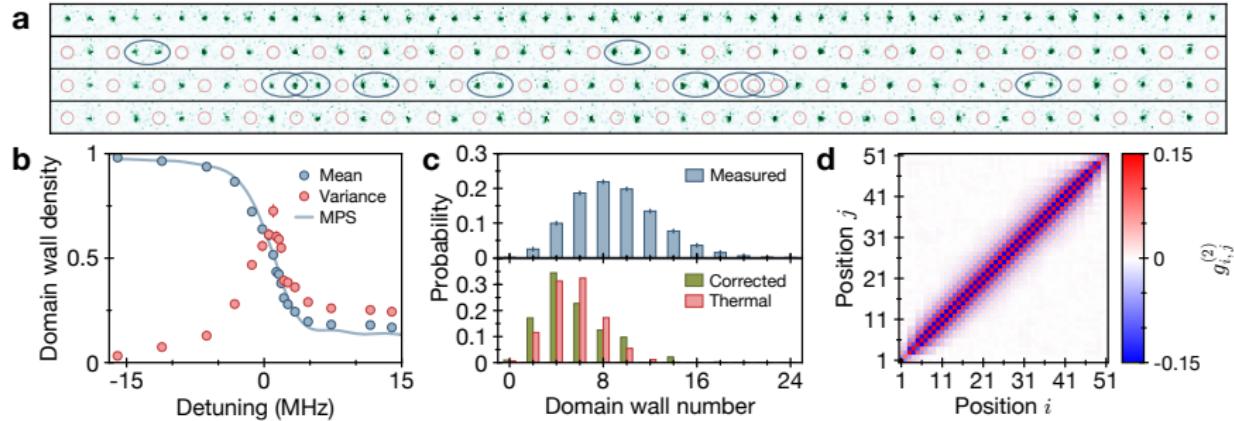
Oscillations of many-body state with 13 atoms



Bernien et al., Nature 551, 579 (2017)]

Many-body physics with Rydberg atoms

Phase transition with 51 atoms

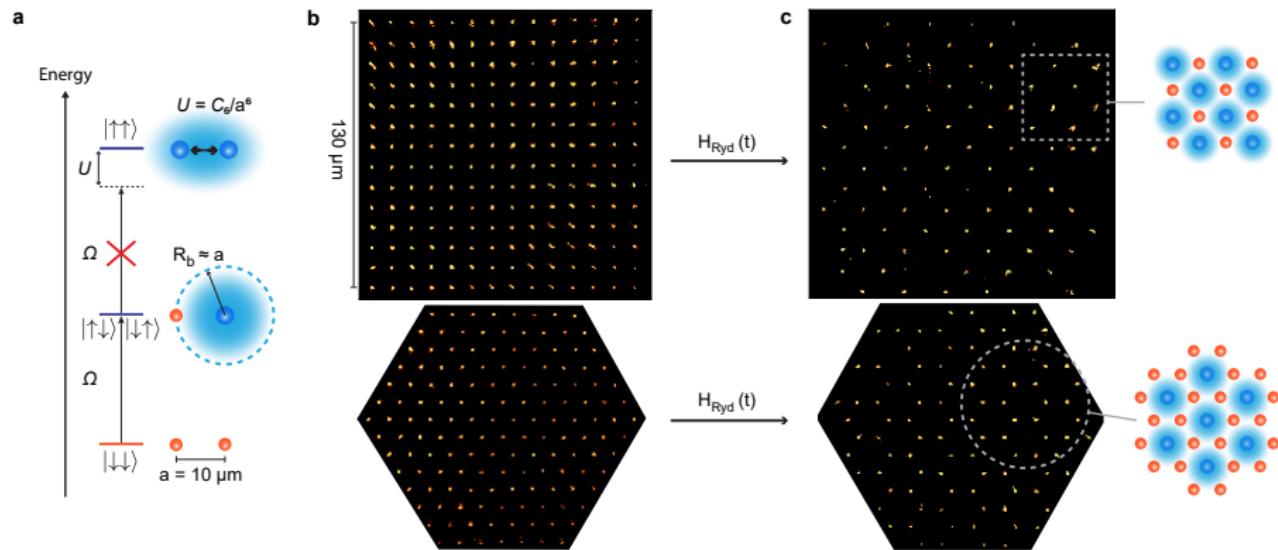


[H. Bernien et al., Nature **551**, 579 (2017)]

Many-body physics with Rydberg atoms

Recent results: more than 200 atoms

*antiferromag
phase!*

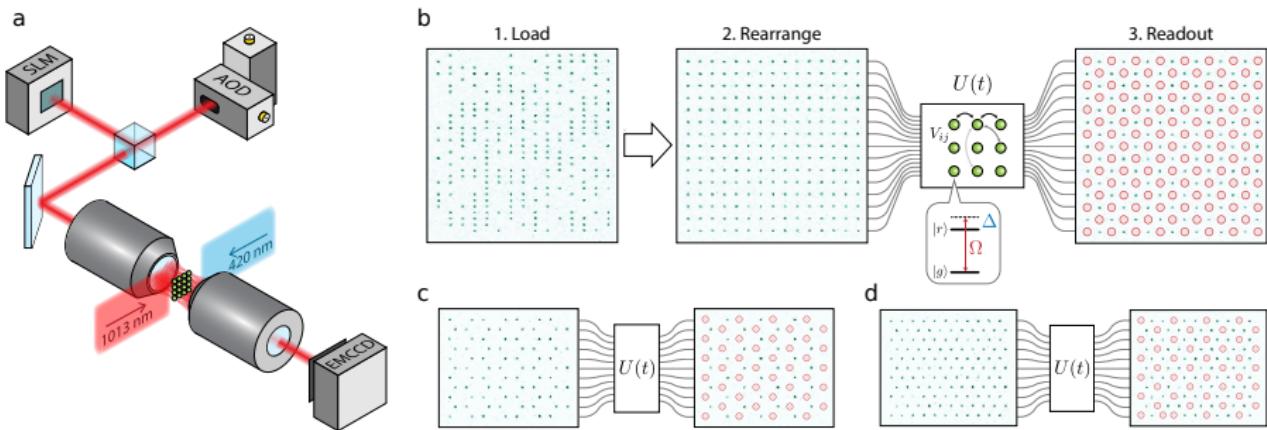


[P. Scholl et al., arXiv:2012.12268 (2020), Browaeys group]

Many-body physics with Rydberg atoms

Recent results: more than 200 atoms

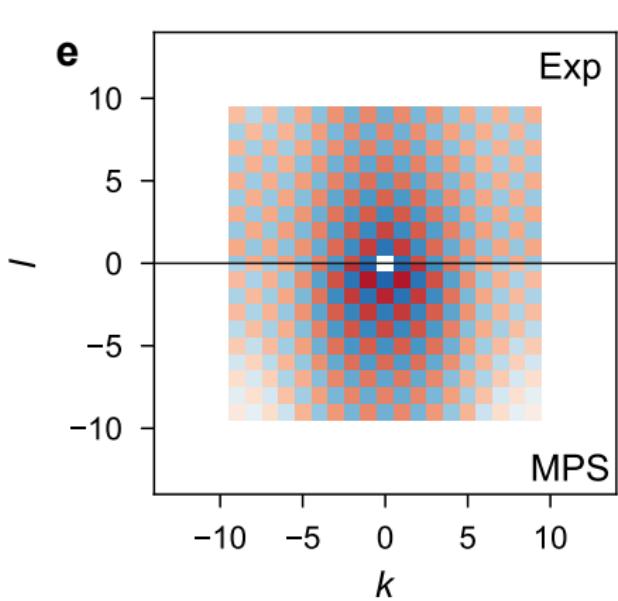
AF.



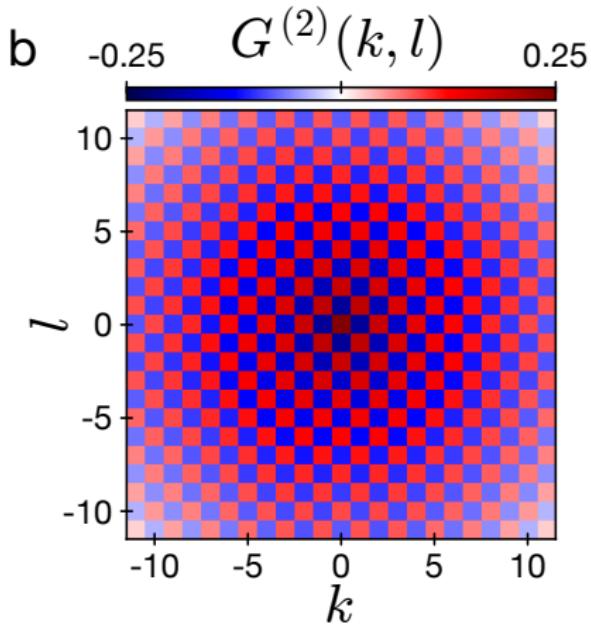
[S. Ebadi et al., arXiv:2012.12281 (2020), Lukin group]

Many-body physics with Rydberg atoms

Observation of the antiferromagnetic state



[P. Scholl et al., arXiv:2012.12268 (2020),
Browaeys group]



[S. Ebadi et al., arXiv:2012.12281 (2020), Lukin group]

Bibliography for Lecture 4

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