Atom interferometry

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Quantum metrology and fundamental constants

Introduction
Why using atoms?

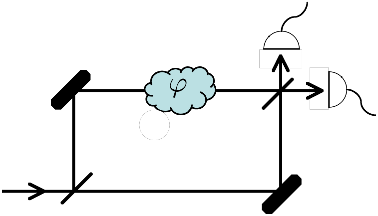
- Light interferometry takes advantage of the wave nature of light to obtain information on the medium where it travels
- Matter wave interferometry gives access to a wider class of information thanks to both external (particle mass) and internal degrees of freedom
- Different atoms may be used allowing mass comparisons
- Atom cooling and trapping allow very long quantum measurement times, long de Broglie wavelength, high precision may be reached

$\Rightarrow$ Atom interferometry with neutral atoms developed rapidly after the demonstration of laser cooling and trapping and led to important advances in precise measurement.

Introduction

General scheme of an atom interferometer: example of the Mach-Zehnder interferometer.

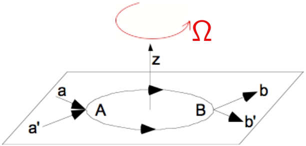
- beamsplitters
- mirrors
- phase object, or physical effect responsible for a phase difference
- detection at the outputs

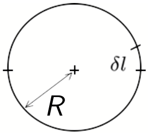


The components of the interferometer may be material objects... or light!

Introduction
Example: gyroscope

Comparison of light and atom interferometry for the measurement of small rotations:





phase difference: $\delta\varphi = 2k\,\delta\ell = 2kR\Omega T = 2\Omega\pi R^2 \dfrac{k}{v}$

light: $k = \frac{\omega}{c}$

$v = c \Rightarrow \frac{k}{v} = \frac{\omega}{c^2}$

atoms: $k = \frac{Mv}{\hbar} \Rightarrow \frac{k}{v} = \frac{M}{\hbar}$

sensitivity: $\dfrac{\delta\varphi_{\mathrm{atoms}}}{\delta\varphi_{\mathrm{light}}} = \dfrac{Mc^2}{\hbar\omega} \sim 10^{11}$!

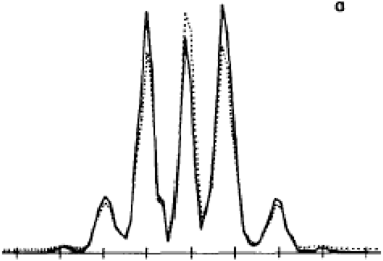
Outline

Outline of the lecture

1. Matter wave diffraction
   - Diffraction by material masks
   - Diffraction by light standing waves
2. Atom interferometry
   - Calculating the atomic phase: example of the double slit
   - Some applications of atom interferometry

Matter wave diffraction

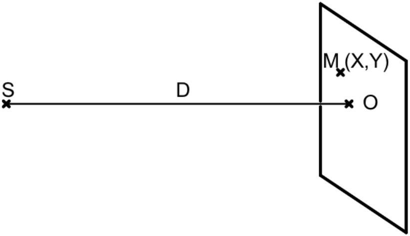
Matter wave diffraction



P. Gould et al. 1986

Huyghens - Fresnel principle

Light waves: $\triangle \mathbf{E} + k^2\mathbf{E} = 0$ for electric field $\mathbf{E}(\mathbf{r}, t)$ and $\lambda = 2\pi/k$
Matter waves: $\triangle\psi + k^2\psi = 0$ with $k^2 = 2ME/\hbar^2$
For a monokinetic atomic beam: $E = \frac{1}{2}Mv_0^2$ and $k = Mv_0/\hbar$
$\Rightarrow$ formally equivalent wave equations; Huyghens – Fresnel principle can be extended to matter waves:



with $T = D/v_0$
and $\hbar k = Mv_0$

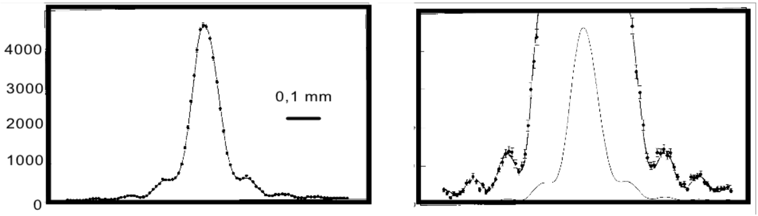
$$\psi(X, Y, D) = \psi_0 \exp\left(\frac{ik(X^2+Y^2)}{2D}\right) = \psi_0 \exp\left(\frac{iM(X^2+Y^2)}{2\hbar T}\right)$$

Diffraction through material masks

A single slit

Matter diffracts through material masks just as light does.

Example: diffraction of a beam of slow neutrons trough a slit of width 93 μm (ILL Grenoble):

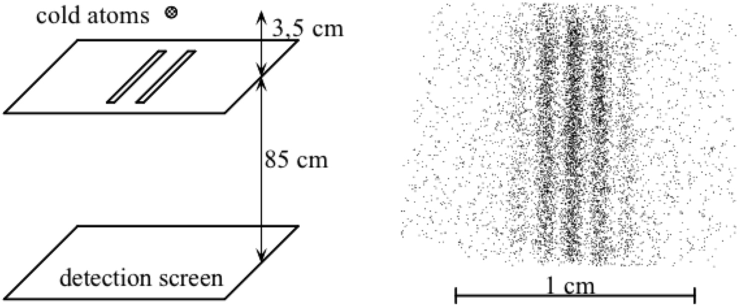


The experiment is in excellent agreement with Huyghens–Fresnel prediction.

Here, $v = 206\,\text{m/s}$, *i.e.* $\lambda = 1.9\,\text{nm}$; a slower beam would given a larger splitting $\Rightarrow$ use cold atoms!

Diffraction through material masks
Double slit

In a double slit experiment, Shimizu *et al.* observed Young fringes formed by metastable neon onto a single atom detector.

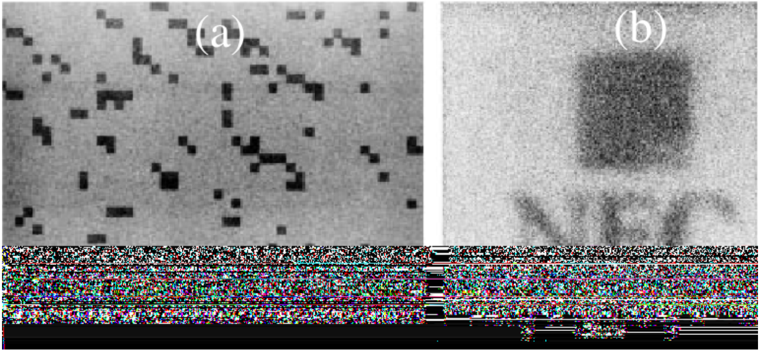


Each spot corresponds to the impact of a single atom onto the detector.

$v = 83\,\text{cm/s} \Rightarrow \lambda \simeq 23\,\text{nm}$ at the double slit mask

Diffraction through material masks
Atom holography

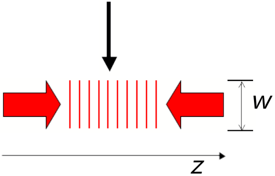
The extension to many holes gives atom holography, where an arbitrary pattern of matter is obtained $\Rightarrow$ Fresnel lenses or even more complex... (Shimizu)



The atomic pattern (b) is the Fourier transform of the mask (a).

Diffraction by a light mask
Thin grating limit

Light may be used instead of material masks, realizing a phase mask with transmission 1.
Ex: a light standing wave as diffraction grating.



Motion along z:

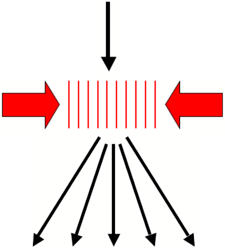
$$H = \frac{P^2}{2M} + U_0 \sin^2(kz)$$

Thin grating approximation: $\frac{w}{v} = T \ll \frac{1}{\omega_{\rm osc}} = \frac{\hbar}{2\sqrt{U_0 E_{\rm rec}}}$
the atoms do not move along z while crossing the light beam.
With an initial state $|p_z = p_0\rangle$:

$$|\psi(T)\rangle = e^{iU_0 T \sin^2(kz)/\hbar}|p_z = p_0\rangle = \sum_n i^n J_n(\frac{U_0 T}{2\hbar})|p_0 - 2n\hbar k\rangle$$

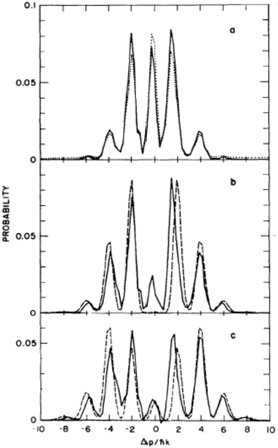
Diffraction by a light mask

Experiments

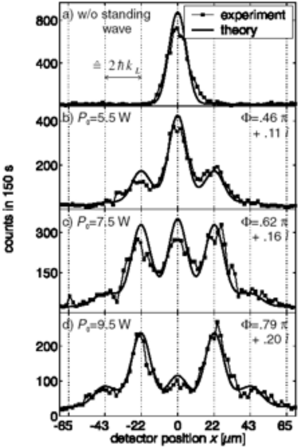


weight:

$$\left| J_n\left(\frac{U_0 T}{2\hbar}\right) \right|^2$$



sodium atoms
Pritchard, 1985



C_{60} molecules
Zeilinger, 2001

Diffraction by a light mask

Energy conservation

Energy change along z (for a small angle, or $p_0 \ll \hbar k$):

$$\Delta E_z = \frac{(p_0 + 2\hbar k)^2}{2M} - \frac{p_0^2}{2M} = 2 p_0 v_{\rm rec} + 4 E_{\rm rec} \simeq 4 E_{\rm rec}$$

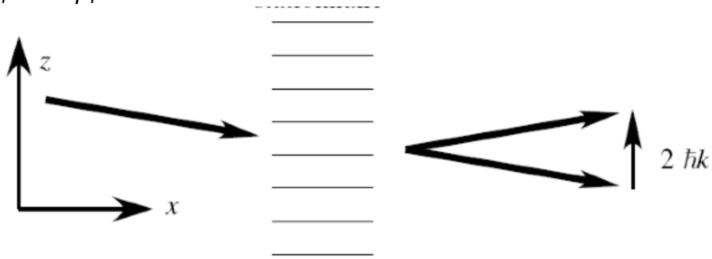
Allowed momentum change along x: $\sim \hbar/w$
$\Rightarrow$ maximum energy change along x:

$$\Delta E_x \leq \frac{(Mv + \hbar/w)^2}{2M} - \frac{1}{2} M v^2 \simeq \hbar v/w = \frac{\hbar}{T}$$

(also valid in pulsed mode).
$\Rightarrow$ possible only if $4E_{\rm rec} < \hbar/T$ or $T < \hbar/4E_{\rm rec}$
Is diffraction possible for a thick grating?

Diffraction by a light mask
Bragg diffraction

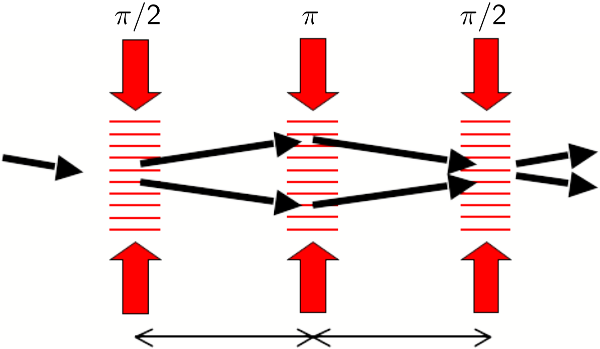
Energy and momentum are conserved for particular initial momenta $p_0 = (2n-1)\hbar k$ along z:
$\Rightarrow p_f = -p_i$



zeroth order: Rabi oscillations between $|p_z = -\hbar k\rangle$ and $|p_z = -\hbar k\rangle \Rightarrow$ beamsplitter with adjustable weights!

Application to atom interferometry
A Mach-Zehnder interferometer

Example of use: a $\pi/2 - \pi - \pi/2$ scheme for implementing a Mach-Zehnder interferometer.



$\Rightarrow$ ideal for building an inertial sensor: measurement of g (put it vertically) or the Earth's rotation (put it horizontally)!

Atom interferometry

Atom interferometry

Path integral formalism

How to calculate the relative phase between two arms of an atom interferometer?

The probability amplitude for a particle to travel from $A(\mathbf{r}_a, t_a)$ to $B(\mathbf{r}_b, t_b)$ is the Feynmann propagator

$$K(\mathbf{r}_b, t_b; \mathbf{r}_a, t_a) = \sum_{\Gamma} e^{iS_\Gamma/\hbar}$$

The sum is over all paths from A to B. The wave function at B is

$$\psi(\mathbf{r}_b, t_b) = \int K(\mathbf{r}_b, t_b; \mathbf{r}_a, t_a)\psi(\mathbf{r}_a, t_a)\, d\mathbf{r}_a$$

and the action S_Γ is deduced from the Lagrangian

$$S_\Gamma = \int_{t_a}^{t_b} \mathcal{L}\left(\mathbf{r}(t), \dot{\mathbf{r}}(t), t\right) dt$$

Path integral formalism

A nice rule:

For a Lagrangian at most quadratic in $\mathbf{r}$ and $\dot{\mathbf{r}}$, K is deduced from the classical action: $K \propto e^{iS_{\rm cl}/\hbar}$

Why?

Summation over all paths $\sum_\Gamma e^{iS_\Gamma/\hbar}$: only stationary phase contributes significantly

$\Rightarrow$ keep paths Γ minimizing the action, *i.e.* classical trajectories

Example of use:

- free particle: $\mathcal{L} = M\dot{r}^2/2$
- particle in gravitational field: $\mathcal{L} = M\dot{r}^2/2 - Mgz$
- particle in an harmonic trap: $\mathcal{L} = M\dot{r}^2/2 - M\omega_0^2 r^2/2$
- particle in a rotating frame:
  $\mathcal{L} = M\dot{r}^2/2 + M\dot{\mathbf{r}}\cdot(\mathbf{\Omega}\times\mathbf{r}) + M(\mathbf{\Omega}\times\mathbf{r})^2/2$

Example: the double slit pattern

Let us calculate the interference pattern for the double slit system in the gravitational field (Shimizu, 1996) in a 2D approach: ▸ figure
slits: $x = \pm d$ and $z = 0$; detector: $z = -H$; initial velocity: $-v_0$.

- Lagrangian: $\mathcal{L} = M(v_x^2 + v_z^2)/2 - Mgz$
- Trajectory $(x_a, z_a, t_a = 0) \longrightarrow (x_b, z_b, t_b = T)$:

$$\begin{cases} v_x = \dfrac{x_b - x_a}{T} = \dfrac{\Delta x}{T} \quad \text{is constant} \\ v_z(t) = v_z(0) - gt \quad \text{with} \quad v_z(0) = \dfrac{z_b - z_a}{T} + \dfrac{1}{2} gT \\ z(t) = v_z(0)t - \frac{1}{2}gt^2 \end{cases}$$

- The classical action is

$$S_{\rm cl} = \int_0^T \frac{1}{2} M \left(v_x^2 + v_z^2(t) \right) - Mgz(t)\, dt$$

Example: the double slit pattern

After integration, we obtain (exercise...):

$$S_{\rm cl} = \frac{1}{2} M \frac{\Delta r^2}{T} - \frac{1}{2} Mg(z_a + z_b)T - \frac{1}{24} Mg^2 T^3$$

where $\Delta r^2 = \Delta x^2 + \Delta z^2$; this term will give the phase deduced from Huyghens – Fresnel principle.

Initial state: $\psi_a(x,z) \propto (\delta(x+d) + \delta(x-d))\,\chi(z) e^{-iMv_0 z/\hbar}$

where $\chi(z)$ is a wave packet centered on $z = 0$ and with central velocity $v_z(0) = -v_0$.

The final state is then:

$$\psi_b(x_b, z_b, T) \propto \int e^{i(S_{\rm cl}(T) - Mv_0 z_a)/\hbar}\, (\delta(x_a + d) + \delta(x_a - d))\,\chi(z_a)\, dx_a\, dz_a$$

Example: the double slit pattern

$S_{\rm cl}(x_a, z_a, T) = S_{\rm cl}(x_a, T) + S_{\rm cl}(z_a, T)$: The integrations over x_a and z_a separate.

$\rightarrow$ Along x_a it gives the two contributions $x_a = \pm d$ creating the fringes.

$\rightarrow$ Along z_a it corresponds to some amplitude $\mathcal{A}(T, z_b)$:

$$\psi_b(x_b, z_b, T) \propto \mathcal{A}(T, z_b) \left(e^{i\dfrac{M(x_b - d)^2}{2\hbar T}} + e^{i\dfrac{M(x_b + d)^2}{2\hbar T}} \right)$$

$$\psi_b(x_b, z_b = -H, T) \propto \mathcal{A}(T, -H) \cos\left(\frac{Md}{\hbar T} x_b\right)$$

The fringe spacing is thus $\dfrac{hT}{2Md}$ — but what is T?

Example: the double slit pattern

$$\mathcal{A}(T,-H) \propto \int e^{\frac{iM}{2\hbar}\left(\frac{(-H-z_a)^2}{T}-gz_aT-2v_0z_a\right)}\chi(z_a)\,dz_a$$

χ is peaked around $z_a = 0$. The integral is very small unless the argument is stationary in z_a around $z_a = 0$.

$\Rightarrow$ true if $2H/T - gT - 2v_0 = 0$ that is $T = \dfrac{\sqrt{v_0^2+2gH}-v_0}{g}$

recover the classical expression for the center of mass.

Final result for fringe spacing D:

$$D = \frac{h}{2Mgd}\left(\sqrt{v_0^2+2gH}-v_0\right)$$

Remark: if $v_0^2 \ll 2gH$: ▸ figure

$$D \simeq \frac{h}{Md\sqrt{2gH}}H = \frac{\lambda H}{d} \quad \text{as in optics...}$$

Some applications

Applications of cold atom interferometry to metrology include:

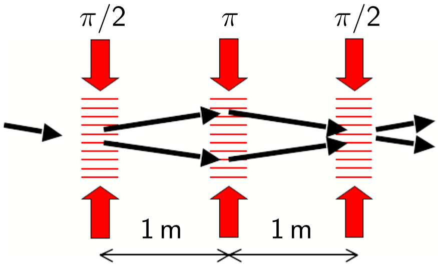
- accelerometers, gravimeter: sensitivity $\propto T^2$
- gradiometer (measurement of G)
- measurement of rotations: Sagnac gyroscopes $\propto LT$
- clocks (internal state) $\propto T$ $\rightarrow$ Sébastien Bize
- measurement of fundamental constants (h/M...) $\rightarrow$ Saïda Guellati and Andreas Wicht

Measurement of Earth's rotation

Back to the atomic gyroscope

$\pi/2 - \pi - \pi/2$ scheme with cesium atoms for an atomic gyroscope (Kasevich, Stanford 1997-2002):

2-photon Raman transitions
$\pi/2 - \pi - \pi/2$ sequence
beam velocity 300 m·s^{-1}
area 0.2 cm^2

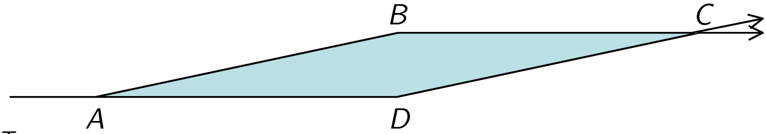


$$\mathcal{L} = M\dot{r}^2/2 + M\dot{\mathbf{r}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}) + M(\boldsymbol{\Omega} \times \mathbf{r})^2/2$$

to first order in Ω:

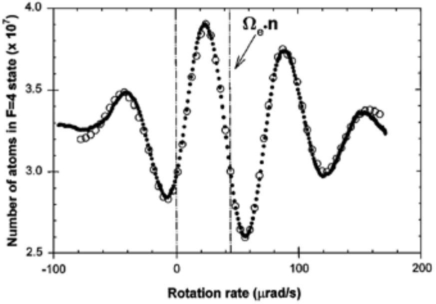
$$S_{\rm cl} = S_0 + M\boldsymbol{\Omega} \cdot \int_0^T \mathbf{r} \times \dot{\mathbf{r}}\, dt \quad \text{with} \quad S_0 = \frac{M\Delta r^2}{2T}$$

Measurement of Earth's rotation



$\int_0^T \mathbf{r} \times \dot{\mathbf{r}}\, dt = \int \mathbf{r} \times d\mathbf{r}$ is the area of $ABCD$

The effect is opposite for ABC and ADC $\Rightarrow \Delta\varphi = 2M\boldsymbol{\Omega} \cdot \mathbf{S}/\hbar$



Results:

sensitivity in 2002:

2×10^{-8} rad·s^{-1}Hz$^{-1/2}$

current sensitivity:

6×10^{-10} rad·s^{-1}Hz$^{-1/2}$

Conclusion

- Laser cooling and trapping greatly improved measurement time, and thus accuracy
- Atom interferometry is well suited for metrology and fundamental tests...
- ...on Earth or in space
- Bose-Einstein condensation is a new tool at the frontier of atomic physics and condensed matter communities

Future prospects: atom interferometry using BEC: 20000 Bloch oscillations obtained in a BEC, current development of on-chip clocks...

Further reading

- Steve Chu's course in Les Houches session LXXII book
- Paul Berman, *Atom Interferometry* (Academic Press, San Diego, 1997).
- lectures of Claude Cohen-Tannoudji at Collège de France 1992-93 and 1993-94 (in french)