Light forces

Light forces are essential in the control of the external degrees of freedom of atoms. They are at the heart of laser cooling and trapping, are necessary for the production of degenerate gases as well as for the realization of atomic clocks.

1. Calculation of the mean force

1.1 System under consideration and useful approximations

We consider a two-level atom[1] with a ground state g, an excited state e of natural line width Γ. The energy difference between e and g is denoted as $\hbar\omega_0$, corresponding to a wavelength λ_0. Its external motion is described by the operators position $\hat{\mathbf{R}}$ and momentum $\hat{\mathbf{P}}$. An example we can keep in mind is an alkali atom excited on its cycling transition $|F, m = F\rangle \to |F+1, m' = F+1\rangle$.

A monochromatic laser source of angular frequency ω is shined onto the atom. Due to the very large photon number in the mode, we describe the field classically under the form

$$\mathbf{E}(\mathbf{r}, t) = \frac{\mathcal{E}_L(\mathbf{r})}{2}\left(\boldsymbol{\epsilon}_L(\mathbf{r})\, e^{-i\omega t - i\phi(\mathbf{r})} + c.c.\right).$$

We consider the atom-field interaction at the dipolar approximation. The dipole operator is

$$\hat{\mathbf{D}} = \mathbf{d}\left(|e\rangle\langle g| + |g\rangle\langle e|\right),$$

where $\mathbf{d}$ is the reduced dipole operator. We define the Rabi frequency as

$$\hbar\Omega_1(\mathbf{r}) = -\left[\mathbf{d}\cdot\boldsymbol{\epsilon}_L(\mathbf{r})\right]\mathcal{E}_L(\mathbf{r}).$$

We assume that Ω_1 is real, which is always possible with a change in the time origin.

1. Give the expression of the recoil velocity $v_{\rm rec}$ defined as the velocity change associated with a single photon absorption or emission. From this expression, define the recoil energy $E_{\rm rec}$, and the recoil (angular) frequency $\omega_{\rm rec}$. Give an estimate of the acceleration which can be applied to the atom with the laser. For numerical estimates, we will use the rubidium data for the $D2$ line $5^2S_{1/2} \to 5^2P_{3/2}$ (see Daniel Steck's data files):

$$h = 6.63\times10^{-34}\ \mathrm{J\cdot s},\ m = 1.44\times10^{-25}\ \mathrm{kg},$$
$$\Gamma = 38\times10^{6}\ \mathrm{s^{-1}},\ \lambda_0 = \frac{2\pi c}{\omega_0} = 780\ \mathrm{nm}.$$

2. Give the expression of the Hamiltonian including the internal and external degrees of freedom, and the atom-field coupling in the rotating wave approximation (RWA).

3. The typical time scale for the evolution of the internal variables is set by Γ^{-1}. The time scale for the external variables can be estimated as the time needed for an atom accelerated by the laser and initially on resonance to get out of resonance due to the Doppler shift. Compare these time scales.

4. We make a semi-classical approximation: the external degrees of freedom are described classically, through their mean position $\mathbf{r} = \langle\hat{\mathbf{R}}\rangle$ and momentum $\mathbf{p} = \langle\hat{\mathbf{P}}\rangle$, whereas the internal degrees of freedom obey quantum equations. This is valid if the fluctuations of the external operators are small in front of the typical length scale and momentum scale of the interaction. Write the conditions which must be fulfilled, and deduce a necessary relation between the atomic characteristics. In the following, we assume that this relation is verified.

[1] The two-level atom approximation is valid if the detuning $\delta = \omega - \omega_0$ is such that $|\delta| \ll \omega, \omega_0, |\omega - \omega_i|$ where ω_i is the transition frequency between g or e and any other atomic state.

1.2 Calculation of the mean force

1. Write the equation of motion for the position and momentum operators, in the Heisenberg picture. Deduce the formal expression of the mean force. We remark that, under the semi-classical approximation, $\langle \mathbf{E}_L(\hat{\mathbf{R}})\rangle \simeq \mathbf{E}_L(\langle\hat{\mathbf{R}}\rangle) = \mathbf{E}_L(\mathbf{r})$.

2. Using the difference in time scales, show that the mean force can be written in terms of the steady state of the dipole operator.

3. We recall the optical Bloch equations for the components $\rho_{ij} = \langle i|\hat{\rho}|j\rangle$ of the atomic density matrix:

$$\frac{d\rho_{ee}}{dt} = -\Gamma\rho_{ee} - i\frac{\Omega_1(\mathbf{r})}{2}\left(\rho_{ge}\, e^{-i\omega t - i\phi(\mathbf{r})} - \rho_{eg}\, e^{i\omega t + i\phi(\mathbf{r})}\right), \tag{1}$$

$$\frac{d\rho_{ge}}{dt} = \left(i\omega_0 - \frac{\Gamma}{2}\right)\rho_{ge} - i\frac{\Omega_1(\mathbf{r})}{2}\left(\rho_{ee} - \rho_{gg}\right) e^{i\omega t + i\phi(\mathbf{r})}, \tag{2}$$

with $\rho_{ee} + \rho_{gg} = 1$ and $\rho_{eg} = \rho_{ge}^*$. We introduce the new variable

$$\tilde{\rho}_{ge}(\mathbf{r}, t) = u + iv = \rho_{ge}\, e^{-i\omega t - i\phi(\mathbf{r})}$$

where u and v are the real and imaginary part of $\tilde{\rho}_{ge}$, respectively. Derive the equations of motion for u, v and ρ_{ee}.

4. What is the stationary solution for $\tilde{\rho}_{ge}$ and ρ_{ee}? Write it as a function of the saturation parameter

$$s(\mathbf{r}) = \frac{\Omega_1^2(\mathbf{r})/2}{\delta^2 + \Gamma^2/4}.$$

5. For a two-level atom, the Rabi frequency is related to the laser intensity I_L through the relation

$$\frac{\Omega_1^2(\mathbf{r})}{\Gamma^2} = \frac{I_L(\mathbf{r})}{2I_s}$$

where the saturation intensity is[2] $I_s = 2\pi^2\hbar c\Gamma/(3\lambda_0^3)$. Give an estimate for I_s. Comment.

6. Give the expression of the mean force in the RWA for an atom at rest. The result can be split into two terms, arising from the dipole component in phase and in quadrature with the excitation field. Explain why these forces are referred to as the dissipative force or radiation pressure, and the conservative force or dipole force.

7. How does the force change if the atom moves with a velocity $\mathbf{v}$?

8. For which experimental conditions is the radiation pressure the dominant force, and for which does the dipole force dominate?

2. Applications of the light forces

We will consider two important applications of the light forces: the Zeeman slower [1, 2] and optical lattices [3].

2.1 Zeeman slower

1. We consider a laser beam modeled by a plane wave with wave vector $\mathbf{k}_L$. What is the expression of $\phi(\mathbf{r})$? Show that the dipole force vanishes, and give the expression of the radiation pressure for an atom at rest, and for an atom moving with a velocity $\mathbf{v}$. Comment.

2. In the early 70's came the idea that the radiation pressure could be used to slow down an atomic beam with initial velocity $\mathbf{v}_0$ with a counter propagating beam. Why is the Doppler effect a problem to reach this goal?

[2] We can also write this relation as $\frac{3\lambda_0^2}{2\pi} I_s = \hbar\omega_0 \frac{\Gamma}{2}$: the saturation intensity is the maximum scattered power divided by the resonant absorption cross section.

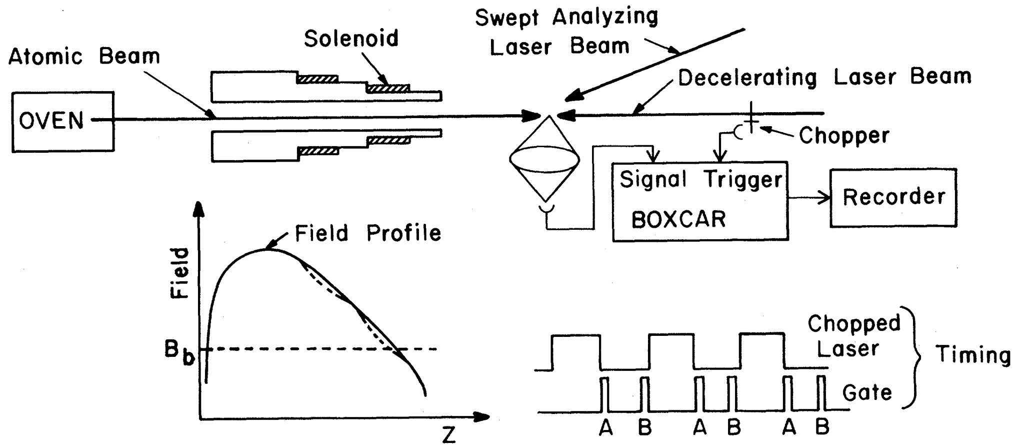

FIG. 1. Schematic diagram of apparatus including optical and electronic arrangement. A plot of B vs z, with and without the turns in the small shaded area of the solenoid, is shown. Note the regions of large dB/dz (steep slope on broken curve). The timing of the boxcar gates A and B relative to the chopping of the decelerating laser is shown at the lower right-hand side.

Figure 1: First setup of a Zeeman slower. Figure from [1].

3. In order to compensate for the Doppler effect, W.D. Phillips and H. Metcalf used a solenoid producing an inhomogeneous magnetic field along the atomic trajectory [1], see Fig. 1. The Zeeman effect shifts[3] the atomic transition by an amount proportional to the magnetic field modulus $B(z)$: $\omega_0'(z) = \omega_0 + \gamma B(z)$. Derive the magnetic field profile which must be used in order to ensure the resonant condition for an atom all the way from its initial velocity to zero velocity.

4. Numerical estimate: calculate the length necessary to bring sodium atoms prepared at an initial velocity of $v_0 = 1000$ m·s^{-1} to rest. Estimate the maximum magnetic field in the solenoid. Its length in Phillips' experiment is 60 cm. Comment. We give for sodium

$$m = 3.82 \times 10^{-26} \text{ kg}, \Gamma = 61.5 \times 10^6 \text{ s}^{-1},$$
$$\lambda_0 = 589 \text{ nm}, \gamma/(2\pi) = 14 \text{ GHz} \cdot \text{T}^{-1}.$$

2.2 Optical lattice

We now consider the situation where two counter-propagating laser beams with identical linear polarization and same intensity create a standing wave. This situation is know as an *optical lattice* [3].

1. We wish to get a situation where the dipole force dominates over the radiation pressure. How should the detuning δ be chosen? Give a possible value in the case of rubidium.

2. Give the expression of the dipole potential from which the dipole force derives. What is the expression of the dipole potential in the low saturation limit where $s \ll 1$? Comment.

3. Give the expression of the dipole potential created by the standing wave as a function the position z along the beam axis. We will denote the maximum potential as U_0. Give the value of U_0 as a function of the intensity I_0 of one of the beams.

4. Derive the oscillation frequency ω_z near one of the potential minima at the harmonic approximation, as a function of U_0 and E_{rec}.

2.3 Optical clocks

Optical lattices are used to mimic solid state physics with ultra cold atoms in periodic potentials [3]. Another important application of optical lattices is the achievement of excellent optical clocks, where an atomic optical frequency is probed on trapped atoms, without Doppler shift of the transition.

[3]The transition used by Phillips and Metcalf is the $3S_{1/2}, F = m = 2 \to 3P_{3/2}, F = m = 3$ transition of the sodium atom. The sodium atom is not strictly a two-level system anymore, which allows for the Zeeman splitting. However, with a σ^+ polarized light, the transition considered here is a closed transition and can be modeled with a two-level system.

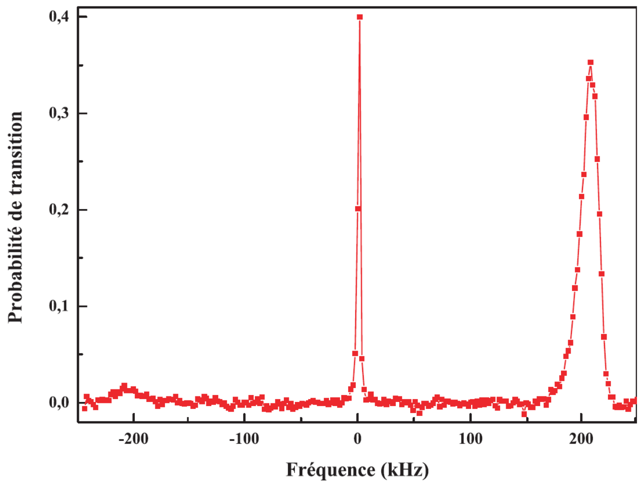

Figure 2: Absorption spectrum of strontium 87 atoms confined in an optical lattice, with $|U_0| = 940\,E_{\text{rec}}$. Figure from [5].

1. The harmonic eigenstates in one of the micro traps are labelled $|n_z\rangle$, with energy $\left(n_z + \frac{1}{2}\right)\hbar\omega_z$. We assume that the excited state $|e\rangle$ undergoes the same potential[4]. Give the energies of the eigenstates $|g, n_z\rangle$ and $|e, n'_z\rangle$. What are the expected transitions of a trapped atom and their frequencies?

2. The Lamb Dicke regime occurs when the ground state in the harmonic trap $|n_z = 0\rangle$ is localized on a scale Δz smaller than the atomic wavelength, which is close to the trapping wavelength: $\Delta z = \sqrt{\frac{\hbar}{2m\omega_z}} \ll 1/k_L$. $k_L \Delta z$ is called the *Lamb Dicke parameter*.

   Give the Lamb-Dicke condition in terms of ω_z and ω_{rec}. Comment on the harmonic expansion done before.

3. Show that the recoil corresponding to a single emission or absorption is now negligible in front of the momentum width of a trapped atom – even in its groundstate. It follows that the probability for changing the external state n_z after a spontaneous emission is very small (of order $k^2\Delta z^2$).

4. Why is this property interesting for frequency metrology?

5. A cloud of atoms trapped in an optical lattice, in the Lamb Dicke regime, is shined with a laser close to the atomic resonance. The resulting absorption spectrum is shown on Fig. 2. Could you expect this spectrum? Deduce the oscillation frequency ω_z from the data. The parameters where $T = 3\ \mu$K and $|U_0| = 940\,E_{\text{rec}}$. The recoil energy for strontium 87 is $E_{\text{rec}}/h = 3.46$ kHz. Comment.

6. How could we generalize this optical lattice to three dimensions?

7. What would be the spectrum like if all atoms were in the harmonic groundstate?

References

[1] William D. Phillips and Harold Metcalf. Laser deceleration of an atomic beam. *Phys. Rev. Lett.*, 48:596–599, Mar 1982.

[2] J. Prodan, A. Migdall, W.D. Phillips, I. So, H. Metcalf, and J. Dalibard. Stopping atoms with laser light. *Phys. Rev. Lett.*, 54(10):992–995, 1985.

[3] Immanuel Bloch. Ultracold quantum gases in optical lattices. *Nat. Phys.*, 1(1):23–30, 10 2005.

[4] Hidetoshi Katori, Masao Takamoto, V. G. Pal'chikov, and V. D. Ovsiannikov. Ultrastable optical clock with neutral atoms in an engineered light shift trap. *Phys. Rev. Lett.*, 91(17):173005, Oct 2003.

[5] Pierre Lemonde. Optical lattice clocks. *Eur. Phys. Jour. Special Topics*, 172:81, 2009.

[4] It is possible when the whole level structure is taken into account, the excited state being shifted due to interactions with higher levels. It happens for certain atoms at a given frequency, called the 'magic frequency'. If the magic frequency is used for trapping, the transition between g and e is unperturbed. This idea of Katori [4] was the starting point of optical clocks [5].