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Atoms dressed by an electromagnetic field

I. Fluorescence in the strong coupling regime

We consider a two-level atom, ground state $|g\rangle$ and excited state $|e\rangle$, interacting with a strong monochromatic electromagnetic field at frequency ω: $\Omega_1 \gg \Gamma$, where Ω_1 is the Rabi frequency characterizing the coupling between the atom and the field, and Γ is the inverse life time of the excited state $|e\rangle$. We will study the fluorescence spectrum of the atom in this strong coupling limit. The excitation field is a strong laser field, close to the atomic resonance at ω_0. We are interested only in the atomic internal states and we won't consider the external state in this problem.

1 Laser field quantization

In a first step, we completely neglect the finite life time of the excited state ($\Gamma = 0$). Although the excitation field is classical in the sense that the mean photon number $\langle N\rangle$ interacting with the atoms is very large, and so is its dispersion ΔN, its relative fluctuations $\Delta N/\langle N\rangle$ are negligible and we will gain a deeper insight in the coupling by using a quantized description of the field. This will make much clearer the interpretation of the fluorescence spectrum. The laser field is thus in a coherent state $|\alpha\rangle$, where we chose $\alpha \in \mathbb{R}$.

We start from the expression of the classical field of the plane wave of polarization $\boldsymbol{\epsilon}$ at fixed position:

$$\mathbf{E}(t) = \mathcal{E}_{\rm cl}\boldsymbol{\epsilon}\, e^{-i\omega t} + c.c.$$

1. Give the relation between α and $\langle N\rangle$ and ΔN.
2. Show that the quantum field can be described as follows:

$$\hat{\mathbf{E}} = \mathcal{E}_0 \left(\boldsymbol{\epsilon} a + \boldsymbol{\epsilon}^* a^\dagger\right) \tag{1}$$

   and give the relation between $\mathcal{E}_{\rm cl}$ and $\mathcal{E}_0$.
3. We recall the dipole operator $\hat{\mathbf{d}} = \mathbf{d}\hat{\sigma}_+ + \mathbf{d}^*\hat{\sigma}_-$ where $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$. Give the expression of the atom-field Hamiltonian, introducing the single photon Rabi frequency Ω_0. Relate it to the average Rabi frequency of the coherent state Ω_1.
4. Write the full Hamiltonian including the atomic internal energy, the quantum monochromatic field and the interaction term.

2 Uncoupled states

1. In the absence of coupling (for $\Omega_0 = 0$), write the eigenstate of the {atom + photons} system and their energy.
2. Show that, for a near resonant frequency, they organized into multiplicities $\mathcal{E}_N$ of dimension 2. Give a sketch of the levels.

3 Dressed states

Now, the coupling is non zero.

1. Identify the states of the uncoupled basis which are now coupled by the field. Do they belong to the same multiplicity?
2. Give their coupling amplitude. We recall that the photon number is very large as compared to 1.
3. Give an estimation of the energy shift induced by the two types of coupling, to lowest order in the perturbation theory. Under which assumption can we restrict ourselves to a given manifold?
4. Diagonalize the Hamiltonian inside a given manifold $\mathcal{E}_N$ and give the expression of the dressed states $|\pm, N\rangle$ and their energies $E_{\pm,N}$. How does the state evolve in a frequency sweep from a large and negative detuning to a large and positive detuning?
5. Calculate the energy in the limit of a large detuning. Comment.
6. Why is the adiabatic sweep an interesting way to transfer the population from $|g\rangle$ to $|e\rangle$ in the presence of inhomogeneous broadening?

7. Give the condition for an adiabatic following of the dressed state during the frequency sweep. We recall from the adiabatic theorem that the overlap of $\partial_t |1\rangle$, the variation of an adiabatic state $|1\rangle$, with any other adiabatic state $|2\rangle$ must remain very small as compared to the frequency splitting between the states:

$$|\langle 2|\,\partial_t\,|1\rangle| \ll \frac{|E_1 - E_2|}{\hbar}.$$

4 Finite life time

We now take into account the finite life time Γ^{-1} of the excited state.

1. To which state will $|+, N\rangle$ and $|-, N\rangle$ decay due to spontaneous emission?
2. What are the expected transition frequencies? Why is this fluorescence called the Mollow triplet?
3. What are the corresponding rates?
4. Calculate, from rate equations, the steady state populations in the states $|+\rangle$ and $|-\rangle$.
5. Give the result in the limit where $|\delta| \gg \Omega_1$ and comment.
6. Same question in the limit where $|\delta| \ll \Omega_1$.

II. Radio-frequency adiabatic potentials

In this problem, we will consider the dressing of atoms with a radio-frequency (rf) field. This technique is used to tailor trapping potentials for atoms in a dressed state, using a position dependent magnetic field [1–4]. The example we can keep in mind is the rubidium 87 atom in its $F = 1$ state, in a static field of about $B_0 = 100\ \mu$T, in the presence of a rf field of a few hundred kHz up to 10 MHz, produced by an alternative current circulating in a loop. We recall the Landé factor $g_F = 1/2$ in the ground state of rubidium such that $g_F \mu_B/\hbar = 2\pi \times 7$ GHz·T^{-1}. We will assume for simplicity that $g_F > 0$.

1 Interaction between a spin and a classical magnetic field

1.1 Static magnetic field

We consider an atom with a total spin $\hat{\mathbf{S}}$. Recall the eigenstates $|m\rangle_z$ and the energies of the spin projection $\hat{\mathbf{S}} \cdot \mathbf{e}_z$ in a static magnetic field $\mathbf{B}_0 = B_0\,\mathbf{e}_z$. Recall the effects of the $\hat{S}_\pm$ operators on the states, where

$$\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y. \tag{2}$$

1.2 Spin in a rf classical field

In this part, we describe the rf field as a classical field. In a first step, we consider atoms with a spin S in a homogeneous magnetic field $\mathbf{B}_0$. The direction of the static field is chosen as quantization axis z: $\mathbf{B}_0 = B_0\,\mathbf{e}_z$.

1. Explain why, at rf frequencies, spontaneous emission can be ignored.
2. Give the corresponding Hamiltonian for the spin in the presence of a static field $B_0\,\mathbf{e}_z$ and a homogeneous rf field $\mathbf{B}_1(t)$.

The classical rf magnetic field can be written very generally as

$$\mathbf{B}_1 = B_x e^{-i\omega t}\,\mathbf{e}_x + B_y e^{i\phi} e^{-i\omega t}\,\mathbf{e}_y + B_z e^{i\phi_z} e^{-i\omega t}\,\mathbf{e}_z + c.c. \tag{3}$$

3. Explain why the z component of the rf field doesn't couple the $|m\rangle_z$ state to another spin state. We will discard this term[1] from now on.
4. We introduce the spherical basis $(\mathbf{e}_+, \mathbf{e}_-, \mathbf{e}_z)$:

$$\mathbf{e}_+ = -\frac{1}{\sqrt{2}}\left(\mathbf{e}_x + i\mathbf{e}_y\right) \qquad \mathbf{e}_- = \frac{1}{\sqrt{2}}\left(\mathbf{e}_x - i\mathbf{e}_y\right). \tag{4}$$

   Give the expression of $\hat{\mathbf{S}} \cdot \mathbf{e}_\pm$.

5. Give the complex projections $B_\pm$ of the rf field amplitude onto this basis.

[1] In fact, misalignment effects of the rf field, where there is a non zero component along the static field, do have an effect, see [5].

6. Write the spin Hamiltonian under the form

$$\hat{H} = \omega_0 \hat{S}_z + \left[\frac{\Omega_+}{2} \, e^{-i\omega t} \, \hat{S}_+ + \frac{\Omega_-}{2} \, e^{-i\omega t} \, \hat{S}_- + h.c. \right]. \tag{5}$$

   Give the expression of $\Omega_\pm$.

7. Show that, in a frame rotating at frequency ω, and if we apply the rotating wave approximation, the spin Hamiltonian reads

$$\hat{H}_{\mathrm{eff}} = -\delta \hat{S}_z + \left[\frac{|\Omega_+|}{2} \, \hat{S}_+ + h.c. \right] = -\delta \hat{S}_z + |\Omega_+| \hat{S}_x. \tag{6}$$

   $\delta = \omega - \omega_0$ is the detuning from resonance.

8. Write the Hamiltonien under the form:

$$\hat{H}_{\mathrm{eff}} = \sqrt{\delta^2 + |\Omega_+|^2} \left(\cos\theta \hat{S}_z + \sin\theta \hat{S}_x \right). \tag{7}$$

   Comment.

9. Give the associated energies and eigenstates using spin rotation operators. What are the effect of both polarization components? Give the coupling amplitude in terms of B_x, B_y, and ϕ.

2 Adiabatic potentials

2.1 Adiabaticity

We now assume that the magnetic field $\mathbf{B}_0(\mathbf{r})$ is position dependent, in amplitude and in orientation. The rf field remains homogeneous.

1. In a first step, we treat the position $\mathbf{r}$ classically, as an external parameter in the Hamiltonian, and we assume that the atoms will follow adiabatically an eigenstate of the effective Hamiltonian. Give the expression of the adiabatic potential resulting from the interaction between the spin and the rf field.

2. We first consider a static magnetic field with a fixed direction, and with a position-dependent amplitude. Show that the atoms with $m > 0$ are confined to an isomagnetic surface. What is the minimum energy spacing between adiabatic states?

3. Give the expression of the oscillation frequency of this confinement in the harmonic approximation. We will denoted $b'(\mathbf{r}) = |\boldsymbol{\nabla} B_0|$ the local gradient of the magnetic field amplitude at position $\mathbf{r}$.

4. Numerics: We give typical figure of experiments with rf adiabatic potentials: $b' \simeq 5$ T·m^{-1}, $|\Omega_+|/(2\pi) = 30$ kHz, spin $F = 1$ for rubidium 87 where $h/M = 4.6 \times 10^{-9}$ Hz·m^2. Evaluate the oscillation frequency. Comment.

   We relax the assumption of a fixed magnetic field direction. If the direction of the static field varies in space slowly as compared to its amplitude, the previous conclusion remains valid. The change in the field orientation will determine the position of the trap minimum inside the isomagnetic surface.

5. If the atomic position and velocity depend on time, the eigenstates will depend on time through $\theta(t)$. Explain why the adiabatic condition reads

$$|\dot{\theta}| \ll |\Omega_+|. \tag{8}$$

2.2 Example: the double-well trap on an atom chip

Using atom chip, where micro-wires are deposited on a substrates, it is possible to bring atoms very close to the wires and reach strong magnetic gradients. By an appropriate choice of wire configuration, we can create a static magnetic field which is the same as in a Ioffe-Pritchard trap:

$$\mathbf{B}_0(\mathbf{r}) = (B_{\mathrm{min}} + \frac{b''}{2} z^2) \, \mathbf{e}_z + b'(x \, \mathbf{e}_x - y \, \mathbf{e}_y) = B_z(z) \, \mathbf{e}_z + b'(x \, \mathbf{e}_x - y \, \mathbf{e}_y).$$

1. What are the isomagnetic surfaces in this configuration?

2. We add an rf field, linearly polarized along x: $\mathbf{B}_1(t) = B_1 \, e^{-i\omega t} \, \mathbf{e}_x$. Explain qualitatively how the coupling strength $|\Omega_+|$ varies along the isomagnetic surface.

3. Deduce that this configuration realizes a double-well potential [6], see Fig. 1. How could we change the distance between the two wells?

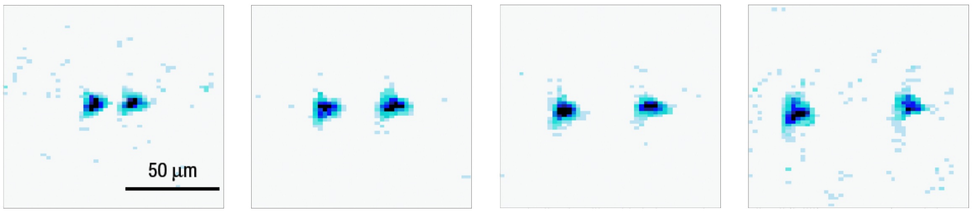

Figure 1: First experimental realization of the rf double well trap [6]. Two Bose-Einstein condensates are trapped in the double well, and imaged along the z direction.

3 Spectroscopy of the dressed states

We now go back to the dressed state picture, for the spin + field system, in the rotating wave approximation.

1. Recall the expression of the dressed states $|\tilde{m}\rangle$ and their energy.
2. We admit that using a second, weak rf field, it is possible [3] to couple two dressed states $|\tilde{m}\rangle$ and $|\tilde{m}'\rangle$. Explain why we must have $|m - m'| \leq 1$.
3. Give the possible transition frequencies.
4. Figure 2 shows rf spectra of rubidium atoms confined in an rf-dressed adiabatic potential. Deduce the Rabi frequency of the rf field from the data.

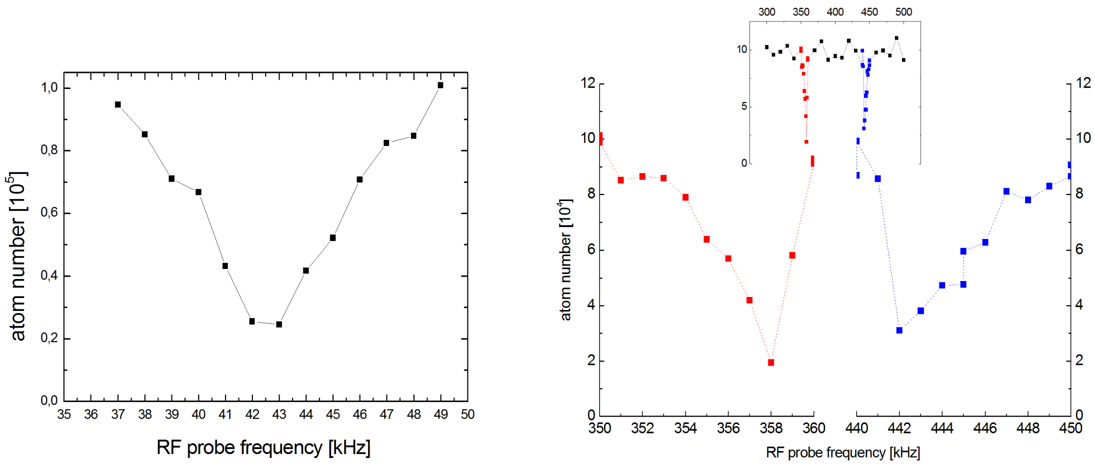

Figure 2: rf-spectroscopy of atoms confined in an adiabatic potential produced by a homogeneous rf field at frequency 400 kHz together with a quadrupole static magnetic field. Spin flips induced by a weak probe produce atom losses, which are recorded as a function of the probe frequency. Losses are observed around 42 kHz, 358 kHz and 442 kHz. Data taken at LPL.

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